

# The influence of network topology and social preference on diffusion processes

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## Abstract

Within the field of innovation diffusion much empirical studies have been conducted on the factors that influence the propagation of new ideas and products. From the natural sciences percolation theory has been used as a starting point to explore the dynamics of innovation diffusion, in particular the occurrence of hits and flops (Solomon *et al.*, 2000). Whereas the latter model is based on a regular network connecting individual consumers, and assumes that consumers have only individual preferences, innovation diffusion theory, as well as empirical data, suggests that consumers differ concerning the number of contacts they have and the degree to which social preferences determine their choice to adopt. To test the impacts of these assumptions on the simulated diffusion dynamics, we replicated the Solomon *et al.* (2000) model, and experimented with scale free networks and social preferences. Results indicate that network shape and social preferences have large impacts on the chances that an innovation either becomes a hit or a flop. To increase the empirical validity of simulated diffusion dynamics we suggest assessing the network structure between consumers as well as the social relevance of the markets.

## Keywords

Innovation diffusion, percolation, social networks, preferences.

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## Introduction

The dispersion of new products, practices and ideas in a population is the basic process underlying societal change. In understanding the processes of societal change, much researchers have focussed at the factors that determine the speed and degree with which new products, practices and ideas propagate through a society. In the social sciences this process is addressed as ‘innovation diffusion’, and the phenomenon has been studied widely using field data. The innovation diffusion theory as introduced by Rogers (1962) is the most frequently cited publication in this field. Rogers states that the cumulative number of adopters typically follows an S-shaped curve in case an innovation succeeds. The S-curve starts to rise slowly when the first *innovators* adopt to the innovation. Following that, the cumulative number of adopters rises somewhat faster due to the *early adopters*. The curve is at its steepest when the *early majority* and *late majority* successively adopt to the innovation. The curve increases at a slower rate when the *laggards* adopt slowly to the innovation. Social interaction between the individuals in the population is the key driver of this diffusion process, and may determine if a diffusion succeeds or not.

Within natural sciences the process relating to influencing through a disordered network structure is addressed as ‘percolation’. The basic idea here is that a social network exists, and that information can propagate through the population through social interaction. Obviously this ‘percolation’ concept provides a perspective for modelling the propagation of new products, practices and ideas through a society.

Whereas the social sciences perspective has a strong tradition in empirical studies but less on modelling processes of diffusion, the natural sciences perspective has a strong tradition in modelling processes, but less on empirical studies of the percolation phenomenon. The basic question is if certain individual and network characteristics, which have been empirically found to relate to the diffusion process, really matter, or that the diffusion process can be represented in a sparser model. Therefore we will compare two models focussing at innovation diffusion. The model we will take as a starting point is the social percolation model of Solomon, Weisbuch, De Arcangelis, Jan and Stauffer (2000). This model will be compared with a model in which empirical based assumptions on network structures and social preferences are included.

## The social percolation model

The model of Solomon *et al.* (2000) and Weisbuch and Stauffer (2000) starts with a two dimensional square lattice where agents are situated in the cells. In this regular network a few agents are situated that already adopted. This adoption is being discussed in terms of visiting a new movie. Now the agent that visited the movie will inform his nearest neighbours in the lattice about the quality of the movie ( $q$ ). The agents are heterogeneous concerning their individual preference ( $p$ ). When agent  $i$  receives information from its neighbour that the quality of the movie is above its critical preference level ( $q > p_i$ ), it will visit the movie. In the next time-step the agent  $i$  will function as a source of information and inform its neighbours about the quality of the movie. Agents having a high preference threshold will not visit the movie and will not inform their neighbours ( $q < p$ ). Due to the fixed preferences of the agents this model represents a classical percolation model. A full rational choice perspective assumes that all agents have perfect knowledge on the movie, and hence the proportion of visitors would equal the proportion of agents for who holds that the quality exceeds its preference threshold ( $q > p$ ). The percolation model however demonstrates that when information is propagated through the social network, assuming the preferences are uniformly distributed between 0 and 1, the success of the movie depends on how close its quality matches the percolation threshold ( $p_c$ ). When the quality of the movie is below the percolation threshold, too few people will visit the movie for the information to disperse through the population.

Hence 'islands' of uninformed agents will emerge, and several agents that otherwise would visit the movie ( $q > p_i$ ) will not go. As the movie will not reach its potential public, it will become a flop. When the movie quality is (sufficiently) above the percolation threshold, the information will reach most of the agents, and hence most of the potential public (roughly the fraction  $q$ ) will actually visit the movie. This model clearly demonstrates how percolation effects may affect the chances of a product to become either a hit or a flop. However, the assumptions of a regular network and fixed preferences are strong and not supported empirically.

### **Diffusion, network structures and social preferences**

The diffusion of innovations is a widely studied phenomenon, and much is known about the characteristics of innovators, early adopters, late adopters, early majority, late majority and laggards. One of the aspects on which people differ is the importance of individual preferences versus social preferences. If a majority of friends goes to a movie, some people will also join despite their dislike for the movie, whereas others refuse to join and stick to their personal preferences. From research it is known that early adopters generally appear to weight their personal preferences more, whereas late adopters appear to attach more weight to their social preferences (Rogers, 1995; pp. 263-264). Rogers (1995) emphasises the importance of reaching a certain 'critical mass' of adopters beyond which the innovation will diffuse without much stimulation. The general idea is that when an innovator has adopted, and a critical mass of early adopters is reached (3 to 16%), the innovation will diffuse without much promotion. Because the majority attaches more relative value to social preferences, the more people in their environment already adopted, the more likely it is that they adopt too because the social utility increases. Eventually also people may adopt who have a relative low personal preference for the movie, but because their social environment visits the movie, they will too. It may be the case that these people have been informed early on the movie and decided not to go, but change their mind if the majority of their social environment does. We will introduce a model for diffusion of innovation based both on individual preference and social preference. Whereas the simple percolation model explains flops in terms of the information not reaching potential visitors, our model for innovation diffusion also allows for hits to emerge on the basis of social preferences.

A next issue that may be of importance is the structure of the network. It may be assumed that people are heterogeneous concerning the number of contacts they have. Some people have a lot of contacts, whereas others only a few. Moreover, some people are more expert on certain topics and will advice many others. These people function as hubs in a scale free network, and may have a disproportional large effect on the diffusion of an innovation. In modelling experiments it has been demonstrated that different assumptions concerning the network may have significant effects on the diffusion of knowledge (e.g., Cowan & Jonard, 2004)) and consumption (e.g., Janssen & Jager, 2003)<sup>1</sup>. The critical question we want to

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<sup>1</sup> In a survey we studied the diffusion process of DVD players amongst 92 people (62 male, 30 female, average age = 27.73). Of these 92, people 66 possessed a DVD player. The critical mass has long been reached, as almost 72% of the sample owns a DVD-player. We are primarily interested in the effects of (1) social interaction and of (2) social preference on the diffusion of DVD players. Concerning social interaction we focussed on how many informational contacts people used. We found that the longer one possesses a DVD player, the more often one advices other people on buying a DVD player (Pearsons  $r$  is  $-.242$ ,  $p < .05$ ). Results indicated that the longer one possesses a DVD player, the higher the percentage of contacts with those that also owned a DVD player (Pearsons  $r = .30$   $p < .005$ ). As we found that the longer one possesses a DVD player the more often one replaced an old model for a new one (Pearsons  $r = .36$ ,  $p < .005$ ), they obviously used their contacts to get information when buying their second or third DVD player. The bottom line is that people advising other people more often also report being advised by others more often (Pearsons  $r = -.269$ ,  $p < .01$ ), indicating that the innovators indeed have more contacts used to exchange information. Concerning the social

answer with our model-to-model comparison is “to what extent is the diffusion process dependent on the structure of the social network and the social susceptibility of the agents”. In the next section we will explain the model that will be used to answer this question.

### The innovation diffusion model

As in the Solomon *et al.* (2000) and Weisbuch and Stauffer (2000) model, we will adopt the binary choice metaphor to see or not to see a movie in order to indicate whether a consumer does or does not buy a product or a person does or does not adopt a given idea. In our innovation diffusion model, agents decide according to a simple weighted utility of individual preference and social influence (or social preference):

$$U_{ij} = \beta_i \cdot \frac{q_j^\gamma}{q_j^\gamma + p_i^\gamma} + (1 - \beta_i) \cdot x_j \quad (1)$$

where  $p_i$  is the individual preference of agent  $i$ ,  $q_j$  is the quality of movie  $j$  and  $x_j$  is the fraction of  $i$ 's friends going to see the movie  $j$  and  $\gamma \geq 20$ . The utility has two components: individual preference, local social influence or social preference.  $\beta_i$  weights the two components of the utility of each agent of the population. When  $\beta_i$  is high, agent  $i$  will be very individualistic, and consequently it will be hardly influenced by its neighbours. On the other hand, when  $\beta_i$  is low, agent  $i$  will be very socially influenced and a big part of its utility will depend on what its neighbours will do. At the same time,  $\bar{\beta}$  determines which kind of market is simulated. When  $\bar{\beta}$  is high the population of agents will be more individualistic and when  $\bar{\beta}$  is low the population will be more socially susceptible. Agent  $i$  will decide to see the movie  $j$  if it has been informed about the movie and if:

$$U_{ij} - U_{min,i} > 0 \quad (2)$$

where  $U_{min,i}$  is the minimum level of satisfaction of agent  $i$ .

In this model, we assume:

- Agents are positioned in a social network. The social network is a connected graph where agents are nodes and links between agents are arcs. The graph is connected which means that a path between any couple of agents always exists.
- Information can be passed from agent  $i$  to agent  $j$ , if and only if there is a link between  $i$  and  $j$ .
- The choice of initial innovators is always exogenous and random
- Choices are binary: it exists only one movie and agents decide either to see or not to see it.
- The population of agents is heterogeneous ( $\beta_i$ ,  $U_{min,i}$  and  $p_i$  vary between 0 and 1).
- Spread of information and social influence are separated phenomena. When an agent becomes aware of the existence of movie  $j$ , it decides either to see or not to see it. If it

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preferences effect, we simply measured if people agreed with the statement “when friends/family have something new, I want it too”. The value of this variable, which represents the factor  $\beta$  of our model (social susceptibility), correlated strongly with time of possession. The earlier one bought a DVD player, the lower the value for  $\beta$  (Pearsons  $r = -.423$ ,  $p < .001$ ). On the contrary, data showed that innovators valued the necessity of the product more, indicating that personal preferences plays a stronger role. The longer one owns a DVD-players, the more they agree with the statement “I only buy a new product when it has become necessary” (Pearsons  $r = .32$ ,  $p < .005$ ) and “I find a product has to prove its use before I buy it” (Pearsons  $r = .24$ ,  $p < .05$ ).

These results indicate that people are heterogeneous concerning the number of contacts used to exchange information, and concerning their social susceptibility. Moreover, it appears that these factors are related to the time of adoption: the more contacts one has, the earlier one adopts and the more socially susceptible one is, the later one adopts.

sees the movie, it will inform its first neighbours, otherwise it will not. An agent  $i$  that knows about movie  $j$  decides either to see or not at any time step of the simulation. It may first decide not to see it but later, when a fraction of its neighbours has already seen movie  $j$ , it may decide to see it.

### **Simulations: experiments and results**

To translate the percolation model of Solomon *et al.* (2000) in our innovation diffusion model we set  $\beta_i = 1$ ,  $U_{min,i} = [0, 1]$  and  $q_j = 0.5$ . Using these settings agents have only individual preferences (social preferences are excluded), the minimum utility for adopting is drawn from a uniform distribution ranging from 0 to 1 and the quality of the product is set at 0.5. Finally  $p_i = [0, 1]$  on a uniform range of 0.5. Moving the value of  $\bar{p}$  from 0.25 to 0.75 we will simulate populations of low and high preferences. A regular network is constructed in which 200 agents are represented, each of them having 4 links with friends (Watts and Strogatz, 1998). This network structure is very similar to the regular lattice used in the Solomon *et al.* (2000) model where agents observe their Van Neumann neighbourhoods. In fact, both networks are very clustered which means that in both networks if agent  $i$  knows agent  $j$  and  $i$  knows also agent  $k$ , it is likely that also  $j$  and  $k$  are already connected with another short path not including  $i$ . Although both networks are very clustered, the regular network is more clustered than the regular lattice. More precisely, in the regular network used in the innovation diffusion model, any couple of friends has always two more friends in common and in the regular lattice used in the percolation model if agent  $i$  and agent  $j$  are friends, two other friends of  $i$  know two other friends of  $j$ . Consequently, the capacity of spreading information among a big population of agents is very weak in both network structures.

In the replication experiment we varied (discrete steps of 0.025) the average preference of the agents  $\bar{p}$  from 0.25 to 0.75 and we observed changes in the fraction of agents visiting the movie ( $f$ ). Thus we obtained 21 conditions. For each condition we performed 50 simulation runs. Figure 2 shows how the fraction of agents going to the movie drops down when agents' preferences become higher. The results demonstrate that percolations ( $f > .95$ ) always occur for conditions where the average preference  $\bar{p} < \approx p_c \approx 0.25$ . For these conditions the information will reach all agents and those agents for whom  $U > U_{min}$  will see the movie. When  $\bar{p} > p_c$ , after a certain short time the spreading of information will stop and only a fraction of the agents for whom  $U > U_{min}$  will go to the movie ( $0.0 < f \leq 1.0$ ). For these conditions those agents that decide to not see the movie do not inform their neighbours so that information cannot reach everybody in the network. Consequently a lot of agents that potentially would see the movie do not see it because they do not know about it.

These results as depicted in figure 2 replicate the results of the percolation model (Solomon *et al.*, 2000). A little change of agents' preferences may cause the movie to become a hit or a flop. Furthermore, the results show that the replication of the percolation model yields results that differ from a model where the agents have complete information, and do not depend on their neighbours to obtain information on the quality of the movie. In figure 2 it can be seen that if agents have complete information, the proportion of agents visiting the movie is much larger for lower values of  $\bar{p}$  because everybody is informed and decides according to its individual preference. In figure 2 it is shown that in this complete information model, results reproduce the line  $f = \bar{U}$ , which is the fraction of agents that see the movie when all agents are completely individualistic and always informed.

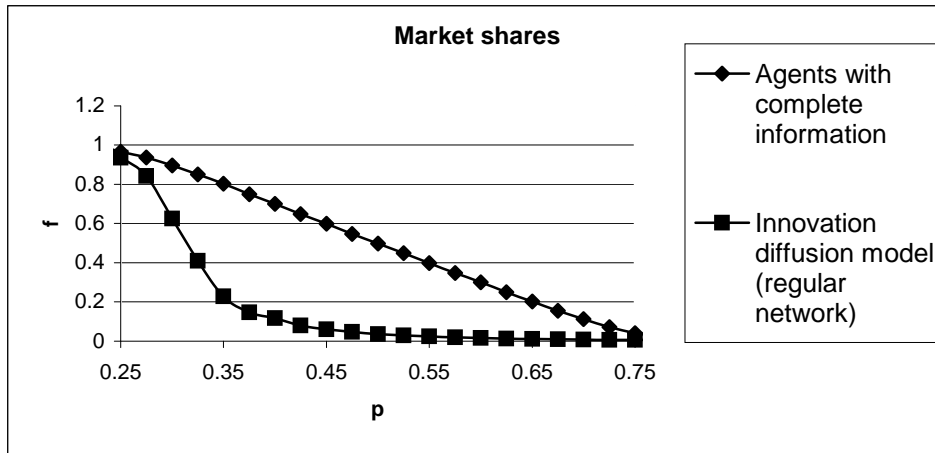


Figure 2. Replication of the Percolation model with the Innovation diffusion model.

However, this replication does not consider other network structures and the local social influence of neighbours. We investigated these two factors using a different network structure and varying  $\bar{\beta}$  value in (1) from 0 to 1.

### Influence of social network structure

Whereas the percolation model is based on a regular lattice, empirical results indicate that people are connected not only locally, but also use more remote links (Bruyn, 2003; Dodds *et al.*, 2003). Moreover, some people have more links than others, a characteristic that has been found to relate to the innovativeness of people. To study how such network assumptions affect the diffusion of innovations, we also formalized a scale free network with cost constraints (Amaral *et al.*, 2000). Whereas in the regular network all agents have exactly four links, in the scale free experiment the agents differ concerning the number of links they have. However, on the average they also have 4 links, and the total number of links is equal in the regular network and in the scale free network. Also for the scale free network structure we performed 50 simulation runs, and each simulation run lasted 60 time steps. In figure 3 we show the market shares for different values of  $p$ .

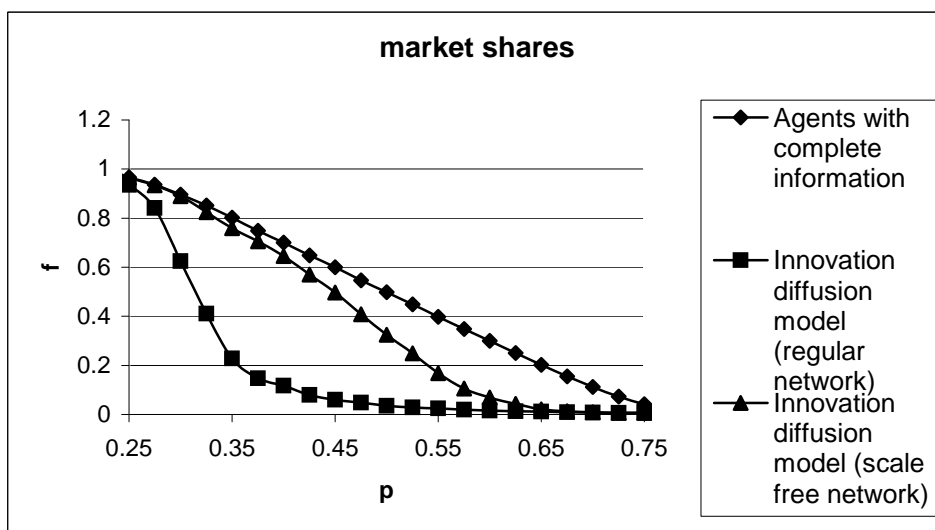


Figure 3. The scale free network model in comparison to the perfect knowledge and the regular network model.

Results indicate that the structure of the network has strong effects on the diffusion process. In case of a scale free network the information spreads much easier through the population, and hence much more potential movie visitors will be informed. For most values of  $p$  the scale free network performs close to the complete information case, thus indicating that a scale free network is very efficient in transmitting information. Only when the preferences of the agents really get high it may be observed that the market shares drop in comparison to the complete information case. This is caused by the effect that the proportion of agents that do not visit the movie increases, and hence they do not inform other agents in their network. Yet it can be seen a large proportion of the potentially interested agents is informed, as in the medium case ( $\bar{p} = 0.5$ ) still about 65% of the potential visitors (50% of the population) is informed and will visit the movie. In the regular network we observe that only about 7% of the potential visitors will visit the movie.

The shape of the network not only affects the degree to which a product diffuses, but also the speed of the diffusion process may differ considerably. In figure 4 we present the average results of 50 runs for the condition where  $p = [0, 0.5]$ , thus involving agents with relative low preferences respect to the quality of the movie ( $q_j = 0.5$ ). For these parameters, in all the 50 repetitions of the run, we observe an almost complete diffusion of the innovation (always  $f \geq 0.9$ ). The figure represents at what time step a given fraction  $f$  of agents has gone to the movie.

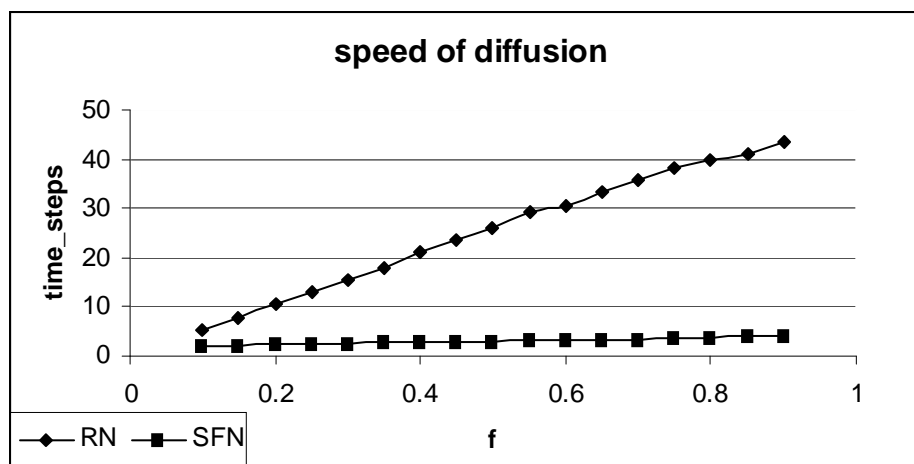


Figure 4. Speed of diffusion in the scale free model in comparison to the regular network model.

We observe that in these favour conditions for diffusion, the scale free network spreads the diffusion much more rapidly than the regular network. On one hand, in the scale free network, an almost complete diffusion is reached just in only 4.9 time steps. This is due to the fact that hubs are informed sooner by early adopters and they can inform easily the rest of the network. On the other hand, the diffusion in the regular network increases linearly. This indicates that also when the fraction of agents seeing the movie is similar for the scale free network condition and the regular network condition, information and diffusion spread faster in the former than in the latter. (I guess this is coherent with other works that have been done and it will be nice to see what happens in the small world networks)

### High social influence versus low social influence

In the previous experiments we only formalised an individual preference, thus replicating the percolation model of (Solomon *et al.*, 2000). However, innovation diffusion theory indicate that consumers also have social preferences (Maybe we should put here some references). Hence we performed a series of experiments in which we varied the average  $\beta$  of

the agents, thus incorporating heterogeneity in the agents concerning the importance of the social preference. The lower  $\bar{\beta}$  is for any agent, the more the behaviour of agents in one's local social network gets in the total utility of the movie. Stated differently, the lower  $\bar{\beta}$  gets, the more the decision of going to the movie depends on what others in one's social network do. We performed experiments for four conditions, setting  $\bar{\beta} = 0.5$  or  $\bar{\beta} = 0.75$ , and using a regular network (RN) or a scale free network (SFN). In figure 5 we represent the average outcomes for 50 model runs for each of the four conditions.

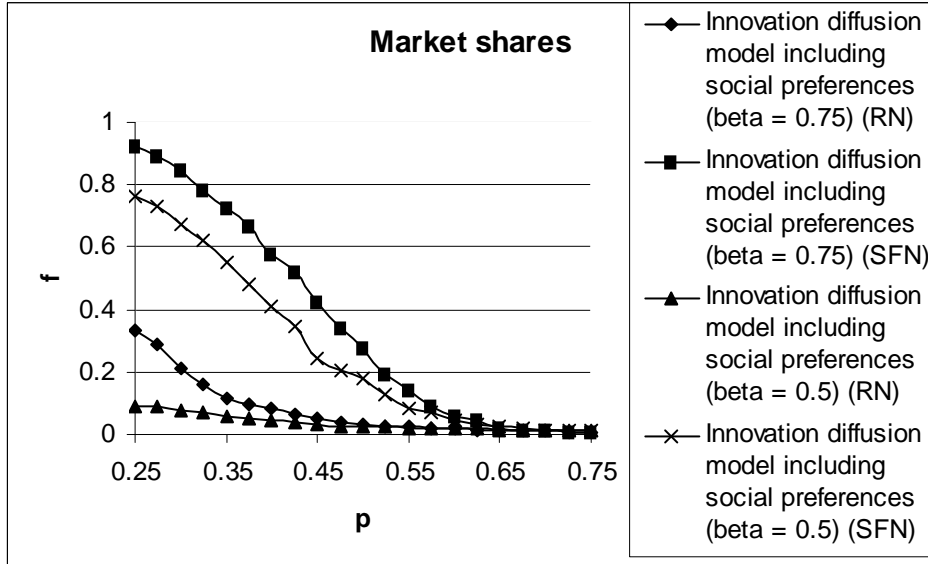


Figure 5. Market shares for different values of  $\bar{p}$  for varying levels of  $\bar{\beta}$  (0,5 or 0,75) and different network types (Regular – RN, Scale free – SFN)

Results indicate that for both conditions of the regular network the diffusion of the innovation is hampered by the social preferences. Obviously, when only a limited number of neighbouring agents adopted, the social preference of adopting oneself will be low, thus decreasing the number of agents that adopt. Hence the market shares drop in comparison to the regular network with only individual preferences (see figure 2, regular network). The stronger the social preference gets (lower  $\bar{\beta}$ ), the fewer agents will adopt. The results for the scale free network show a much larger market share, exceeding the market share obtained with the regular network without social preferences (replication of the percolation model, see figure 2, regular network). Apparently the scale free network performs better in the diffusion of the information, and because more agents will adopt the critical mass increases, thereby also increasing the social preference for the agents that did not adopt at the beginning.

To check this explanation we studied a typical run of a hit with a scale free network ( $p=[0, 0.5]$ ,  $U_{min,i}=[0, 1]$  and  $\beta=[0, 1]$ ). In this condition the innovation was completely diffused in 7 time steps. In figure 6 we show the S curve of the diffusion and in figure 7 we show the number of contacts the agents had against their time of adoption.



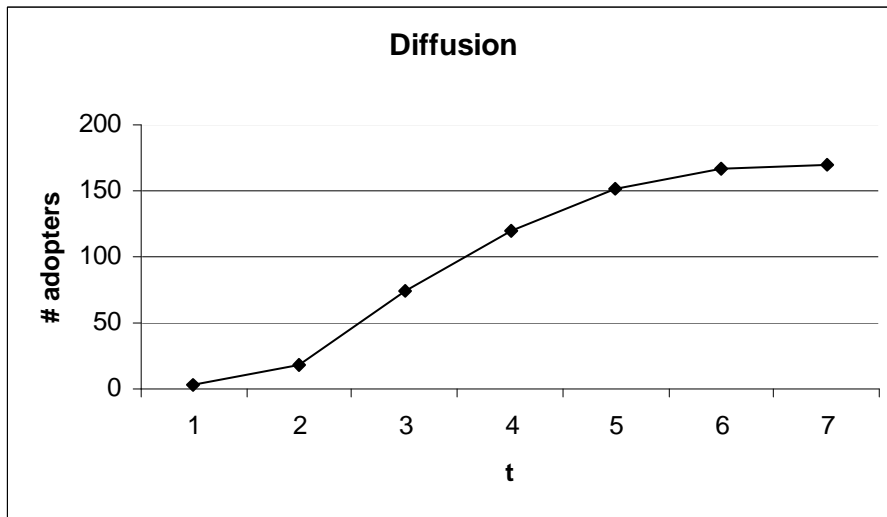


Figure 6. The number of adopters against the time of the diffusion

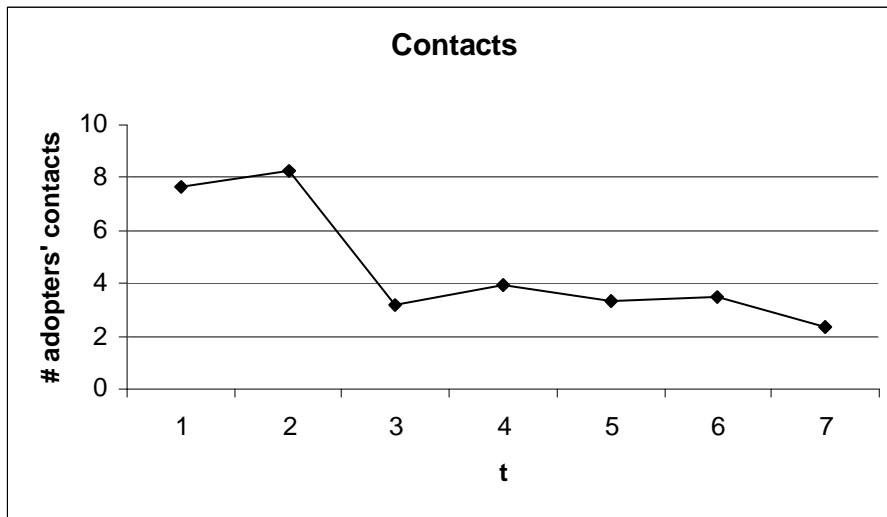


Figure 7. The number of contacts of agents against their time of adoption.

Whereas these results are tentative, and we plan to scrutinize these results for more runs and conditions, the results suggest that indeed the innovators at  $t = 1$  have more contacts with other agents than the agents that adopt later. This indicates that the power of social networks resides in its capacity to spread information very quickly through the hubs in the network.

Next we also checked this run for the  $\beta$  values of the adopting agents for these first 7 time-steps (figure 8).

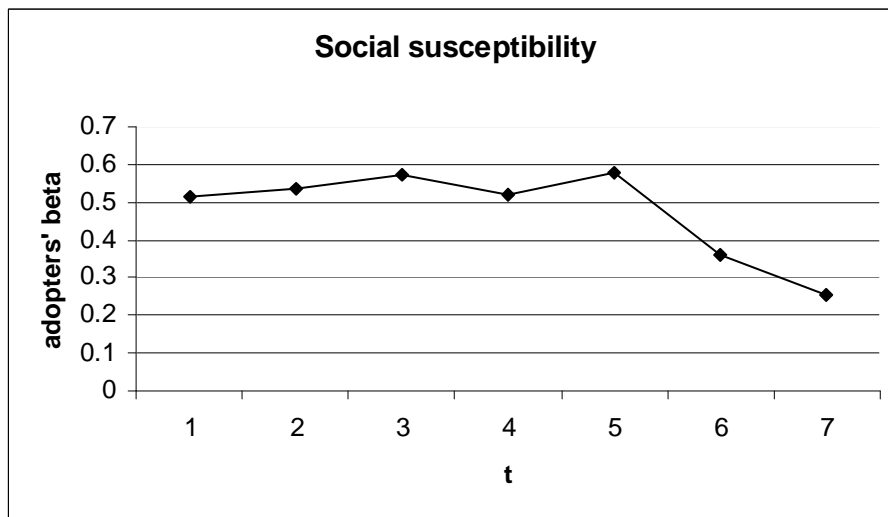


Figure 8. The social susceptibility ( $\beta$ ) of agents against their time of adoption.

These indicative results show that the later adopters have a lower  $\beta$ , and hence are more social susceptible. These agents are more likely to adopt if a sufficient number of other agents in their social network already adopted. Further research is scheduled to explore what types of actors adopt at what moment in the diffusion process for different conditions.

## Conclusions

Using the innovation diffusion model we were able to replicate the results obtained by the percolation model of Solomon *et al.*, 2000. Innovation diffusion theory and empirical data suggest that the regular network as used in the percolation model is not always capturing network structures determining the information spreading in consumer domains. Hence we formalised a scale free network. The results obtained with the scale free network indicated that this type of network is very efficient in the spreading of information, as with a limited number of only 4 links on average the diffusion was almost as complete as in the case of complete information. From these results we conclude that the type of network that connects individual consumers is very important in understanding the degree and speed of the innovation diffusion process.

Including social preferences in the model as suggested by diffusion theory and empirical data, we found that in the regular network the diffusion sharply drops. When consumption is socially relevant, and hardly any one adopted the new product, its diffusion will be smaller the more important the social preference is. The scale free network however demonstrated that even under this condition it is capable to support the diffusion of new products. Apparently the first adopters have a lot of contacts, and succeed in generating sufficient critical mass that other agents will be informed too. These other agents, having a higher social susceptibility, will be more likely to adopt in successive time-steps.

Basically we conclude that hits and flops dynamics by and large depend on the structure of the network connecting individual consumers, and the degree to which their utility of a product includes social preferences. It may be assumed that in many markets some consumers function as hubs informing many other people around them about interesting products. These agents are in innovation diffusion theory addressed as 'change agents' (...). In making a new adoption a hit, it seems often a good strategy to target these hubs in adopting the new product. Once these people have adopted the spread of information may proceed very fast, create a critical mass of adopters, which is subsequently capable of stimulating people with higher social susceptibility to adopt.

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