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Theory and Methodology

Optimization of a non-linear dynamical system for global climate change

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Abstract

We regard the global climate system as a controlled dynamic system, with controls corresponding to economic activities causing emissions of greenhouse gases. Previous optimization studies for climate change have used descriptions of the environmental system which are found to be too unrepresentative of what is known in the scientific community. In this paper an approach is applied which tries to include a more sophisticated model of the environmental system. The resulting continuous dynamic control problem is solved by the application of a set of non-linear optimization techniques to find optimal response strategies to maximize the discounted sum of future consumption while adhering to certain environmental constraints. © 1997 Elsevier Science B.V.

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1. Introduction

Since the beginning of the industrial revolution, the atmospheric concentrations of carbon dioxide (CO_2) and other greenhouse gases such as methane (CH_4), nitrous oxide (N_2O), and chlorofluorocarbons (CFCs) have been increasing substantially. Combustion of fossil fuels for energy, land use changes and the use of CFCs are the main activities responsible for the increase in the concentration of greenhouse gases. These gases are radiatively active in such a manner that increasing concentrations of these gases alter the long-wave radiation from the surface of the Earth and hence may lead to additional warming at the Earth's surface: the *enhanced greenhouse effect.* There is serious concern that human activities may inadvertently change the long term climate of the globe through this enhanced greenhouse effect. Since it is obvious that global climate change may have a significant impact on ecosystems and human societies, an increased amount of effort has been devoted to the study of the dynamics of the climate system during recent years.

The Intergovernmental Panel on Climate Change (IPCC) is placing increasing emphasis on the use of dynamic or time-dependent simulation models to assess the effects of global climate change. At present, various types of scientifically-based computer models describing climate change on a global scale are in use. We will discuss two important streams,

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namely: GCMs (General Circulation Models), and IAMs (Integrated Assessment Models).

General Circulation Models (GCMs) simulate virtually all of the underlying physical and chemical processes on a spatial scale. Coupled Ocean-Atmosphere GCMs possess a three-dimensional structure of the ocean-atmosphere system divided into a number of layers, sub-divided into grid boxes along lines of longitude and latitude. Achieving discreteness via the grid structure enables numerical solutions of the model equations. Models of this type are used, for example, to estimate the equilibrium surface temperature increase which would follow a doubling of carbon dioxide concentration. GCMs, however, require vast amounts of computational time.

Integrated Assessment Models (IAMs) are scientifically-based models which describe the climate system and economic system on a global scale to support policy making. Although IAMs do not describe the complex climate system in detail but rather use simplified versions of specialist models, they can be used interactively to estimate effects of various scenarios and provide a bridge between natural scientists, economists, social scientists and decision-makers. Various modelling frameworks are employed by IAMs: among which are simulation/optimization models and deterministic/stochastic approaches (Dowlatabadi, 1994). A second twofold distinction can be drawn by recognizing that while several models are normative (which is a common approach in economics, e.g.: DICE (Nordhaus, 1992, 1993, 1994), MERGE (Manne et al., 1994), CETA (Peck and Teisberg, 1993)) other models are descriptive (which is common in ecological and climate modelling, e.g.: IMAGE 1.0 (Rotmans, 1990), STUGE (Wigley et al., 1991), ESCAPE (Rotmans et al., 1994)).

Although several IAMs incorporate optimization models for modelling economic activities, integrated optimization approaches for climate change are not widely used. We are only aware of a handful of studies which incorporated climate change impacts in the optimization of climate change response strategies. Nordhaus (1992, 1993, 1994) and Tahvonen et al. (1993) have used simplified model versions of the climate system in their optimization frameworks allowing straightforward application of standard optimization techniques. Nordhaus (1993) argues that "existing [physical] models are, unfortunately, much too complex to be included in economic models". Arguably, the simplified climate system does not deliver an adequate representation of the underlying systems' processes and dynamics. As a consequence, such optimizations are too unrepresentative of what is known of the climate system (Price, 1995). The aim of this paper is not only to show that it is possible to include more sophisticated descriptions of the climate system, an approach which was introduced by Janssen et al. (1995), but also that it makes a significant difference in the derived strategies.

In this paper we employ a mathematical system for climate change OMEGA (Optimization Model for Economic and Greenhouse Assessment) which combines DICE (Dynamic Integrated model of Climate and the Economy) (Nordhaus, 1992, 1993, 1994) and IMAGE 1.0 (Integrated Model to Assess the Greenhouse Effect) (Rotmans, 1990) from which the dynamic system representation of the climate system is borrowed (Braddock et al., 1994; Zapert, 1994), allowing us to use a set of non-linear optimization techniques. DICE is primarily an economic model enlarged by the addition of a oversimplified climate model while IMAGE 1.0 describes the climate dynamics from a natural scientific perspective. By using the economic component of DICE and the climate system component of IMAGE 1.0 we obtain a hybrid model in which we use the strengths of each model to overcome the weaknesses of the other (Filar et al., 1995).

2. The model

2.1. DICE: Dynamic integrated model of climate and the economy

DICE is an optimization model for the economics of climate change based on aspects of optimal growth theory. It is a transposition of Ramsey's model (Ramsey, 1928) of optimal economic growth to climate change policy. Hence, as well as calculating optimal capital accumulation DICE calculates greenhouse gas emission reduction by maximizing the discounted value of utility from consumption.

The model contains both a conventional economic component and a novel climate component. Population growth and technological changes are regarded as being exogenous, while the optimized flow of consumption over time determines accumulation of capital.

In the mathematical formulation of DICE the state variables are represented by $y(t) \in \mathbb{R}^9$ which are dependent on the control variables $z(t) \in \mathbb{R}^2$.

2.1.1. Objective function

The DICE model is designed to model a situation in which the discounted sum of the general level of consumption achieves a maximum. The objective function maximized is:

$$\underset{z_1, z_2}{\text{maximize}} \int_{t=1900}^{2100} l \cdot \ln(y_2) \cdot (1+\rho)^{(1990-t)}, \quad (1)$$

which expressed the discounted sum of utilities of consumption summed over the relevant time horizon. The level of utility or social well-being is expressed in $\ln(y_2)$ whereby y_2 describes the level of consumption per capita, l is the size of population, and ρ is the pure rate of social time preference ($\rho = 3\%$ per annum). The control variables z_1 and z_2 represent gross investment and the rate of emission reduction respectively.

2.1.2. Economics component

Output y_1 is given by a standard constant-returns-to-scale Cobb–Douglas production function in the levels of technology a, capital y_3 and labour which is assumed proportional to l, where labour inputs are proportional to population and γ represents the elasticity of output with respect to capital taken as 0.25. The impact of emission reductions and global climate change on output is represented by the scale factor y_9.

$$y_1 = y_9 \cdot a \cdot [y_3]^{\gamma} \cdot l^{1-\gamma}. \quad (2)$$

Per capita consumption y_2 is defined as the ratio of total consumption, which is equal to economic output minus gross investments, and population:

$$y_2 = (y_1 - z_1)/l. \quad (3)$$

The capital balance equation for the capital stock y_3 is,

$$\mathrm{d}y_3/\mathrm{d}t = z_1 - \delta_k \cdot y_3, \quad (4)$$

where δ_k is the rate of depreciation of the capital stock, 10% per annum, reflecting an average lifetime of capital of ten years on a declining balance method.

The final economic equation represents the fossil and non-fossil emissions amount of CO_2 and CFCs related to economic output:

$$y_4 = [1 - z_2] \cdot \sigma \cdot y_1, \quad (5)$$

where z_2 is the rate of emission reduction and σ the trend in CO_2 equivalent emissions per unit of gross output in the absence of emission controls.

2.1.3. Climate system

The accumulation and transportation of the greenhouse gases in the atmosphere can be expressed as follows:

$$\mathrm{d}y_5/\mathrm{d}t = \beta \cdot y_4 - \delta_m \cdot [y_5 - y_{5,1900}], \quad (6)$$

where y_5 is the concentration of CO_2 in the atmosphere, β is the marginal atmospheric retention rate (the fraction of an emission that remains in the atmosphere in the short run), and δ_m is the rate of transfer from the atmosphere to the oceans.

The relationship between greenhouse gas accumulation and increased radiative forcing, y_6 in W/m^2 is given as:

$$y_6 = 4.1 \cdot [\ln(y_5/y_{5,1900})/\ln(2.0)] + O, \quad (7)$$

where O represents other greenhouse gases such as methane CH_4 and nitrous oxide N_2O.

The increase in globally-averaged temperature in the atmosphere and the upper level of the ocean is expressed as:

$$\mathrm{d}y_7/\mathrm{d}t = (1/R_1) \cdot \{y_6 - \lambda \cdot y_7 - (R_2/\tau_{12}) \cdot [y_7 - y_8]\}, \quad (8)$$

where y_8 is the temperature increase in the deep oceans, R_1 and R_2 are the thermal capacities of the upper layer and the deep ocean respectively, τ_{12} is the transfer rate from the upper layer to the lower layer, and λ the climate sensitivity parameter (°C/W-m^2).

The temperature of the deep ocean is modelled as:

$$\mathrm{d}y_8/\mathrm{d}t = (1/\tau_{12}) \cdot [y_7 - y_8]. \quad (9)$$

2.1.4. Impact on economic growth

The scaling factor y_9 is the ratio of 1 minus the percentage abatement cost and 1 plus the percentage of damage costs:

$$y_9 = (1 - b_1 \cdot z_2^{b_2})/(1 + \theta_1 \cdot y_7^{\theta_2}). \quad (10)$$

The (market) damage costs are quantified as a relation between global temperature y_7 increase and income loss, where θ_1 represents the scale of damage and θ_2 the non-linearity in the damage function. The costs of reducing emissions of greenhouse gases are related to z_2, the fractional reduction of greenhouse emissions, while b_1 and b_2 represent the scale and non-linearity of the cost function.

2.2. IMAGE 1.0: The integrated model to assess the greenhouse effect

IMAGE 1.0 is an integrated modular system incorporating relatively simple models of subsystems assessing the greenhouse effect on a global level (Rotmans, 1990) and has been designed to describe the major cause and effect relationships with respect to climate change. The strength of IMAGE lies in the integration and interfacing of a large number of models drawn from a variety of disciplines. In general, the models used describe only essential processes, although the overall modular design allows for the inclusion of more complex models.

IMAGE was developed as a computer-based simulation model to assess the impact of past, present and future emissions of greenhouse gases on global and regional temperature changes, sea level rise, and a range of other physical and socio-economic variables.

A mathematical system extracting the structure of IMAGE 1.0 as a system of 155 differential equations has been developed to investigate the stability of the simulation model (Braddock et al., 1994; Zapert, 1994) and it is this system that is used in this paper.

The mathematical representation of IMAGE 1.0 is of the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x}(t) = F(\boldsymbol{x}(t)) + \boldsymbol{u}(t) \qquad (11)$$
$$\boldsymbol{x}(t_0) = \boldsymbol{x}_0,$$

where time $t \in [1990, 2100]$, $t_0 = 1990$ is the initial simulation time, $\boldsymbol{x}(t) \in \mathbb{R}^{155}$ is a vector of state variables, $\boldsymbol{u}(t) \in \mathbb{R}^{155}$ is the forcing term and $F(\boldsymbol{x}(t)): \mathbb{R}^{155} \to \mathbb{R}^{155}$ describes the climate system. The solution $\boldsymbol{x}(t)$ as a function of time is a trajectory of Eq. (11). For a detailed description of the formulation of the mathematical system of IMAGE 1.0 we refer to Zapert (1994).

In representing the system components, the state vector $\boldsymbol{x} = \boldsymbol{x}(t) \in \mathbb{R}^{155}$ at time t, is partitioned into the variable groups: carbon contents in the different layers of the ocean ($x_1 .. x_{12}$), atmospheric concentration of CO_2 (x_{13}), carbon contents within the levels of the ecosystems ($x_{14} .. x_{62}$), areas for the ecosystems and their change in use ($x_{63} .. x_{91}$), temperature changes in the various layers of the ocean ($x_{92} .. x_{140}$), surface mean temperature change, x_{141}, concentrations of various greenhouse gases ($x_{142} .. x_{155}$).

The forcing terms represents the human interference in the system and consist of fossil CO_2 emissions, ecosystem transfer rates and other than carbon dioxide greenhouse gas emissions. Implicitly, the forcing term $\boldsymbol{u}(t)$ includes the given scenarios estimation of future population growth, fossil fuel combustion, deforestation, and the technology development in the period 1990–2100 (Rotmans, 1990; Braddock et al., 1994; Zapert, 1994).

2.3. OMEGA (an optimization model for economic and greenhouse assessment)

We propose to combine the economic component of DICE (Eqs. 1–5) and the impact factor (Eq. 10) with the mathematical system formulation of IMAGE 1.0. To be more precise, Eqs. 6–9 from the DICE model are discarded and replaced by the IMAGE 1.0 framework, which means that the emissions of fossil CO_2 (u_{13}) are equal to $y_4(t)$. There is an additional difference in that we will now only consider fossil CO_2 emissions for $y_4(t)$ and therefore adopt a rescaled factor for b_1 based on Nordhaus (1991). Non-fossil CO_2 and CFC emissions are included within the framework of IMAGE. A detailed discussion on OMEGA can be found in Janssen (1995).

3. Optimization methodology

The generic optimal control problem which we can associate with the coupled model has the following form:

$$\underset{z}{\text{maximize}} \int_{t_0}^{t_1} f(x, y, u, z)\, \mathrm{d}t$$

s.t.

$$z \in \mathscr{S} = \left\{ z \in \mathbb{R}^2 : g(x,y,u,z) \leq 0 \right.$$

$$\left. \frac{\mathrm{d}(y,x)}{\mathrm{d}t} = F(y,x) + (u,z) \right\}, \qquad (12)$$

where the objective is to maximize $f(\cdot)$, the discounted sum of consumption, in order to satisfy $g(\cdot)$. The input/control can be split into decision variables of the optimization problem z_1 and z_2 and the remaining input variables $\boldsymbol{u}$ which follow prescribed scenarios as given in Rotmans (1990).

The problem described in (12) is a continuous dynamic control problem. Due to the dimensionality of the problem it may be practically impossible to find an analytical solution. If the time steps are rendered discrete, the problem can be transformed into an ordinary large scale non-linear problem, which we can solve using various numerical optimization techniques, within the limitation of these methods.

For simplicity, the levels of the decision variables are assumed to change linearly over fixed time intervals during the period under consideration (1990–2100). The following time intervals are used: $[T_0, T_1], \ldots, [T_{10}, T_{11}], T_0 = 1990, T_1 = 2000, \ldots, T_{11} = 2100$. The investments and emission control rates are assumed to change in a linear fashion during $[T_k, T_{k+1}]$ where the investments and control rate levels in years T_k and T_{k+1} are denoted by, respectively, $\mu_{1,k}$ and $\mu_{2,k}$. The choice of $\boldsymbol{\mu}_1 = [\mu_{1,k}]_{k=1}^{11}$ and $\boldsymbol{\mu}_2 = [\mu_{2,k}]_{k=1}^{11}$ together with the known value of the initial values $u_{1,1990}$ and $u_{2,1990}$, determine $\mu_{1,k}$ and $\mu_{2,k}$. The $\mu_{1,k}$'s and $\mu_{2,k}$'s can be considered as the decision variables which determine z.

3.1. General non-linear optimization algorithms

The general aim of constrained optimization approaches is to transform the problem into a more straightforward sub-problem which can be solved and used as the basis of an iterative process. This paper considers three constrained optimization approaches: the Penalty Method (PM), Sequential Quadratic programming (SQP) and Sequential Reduced System Programming (SRSP).

3.2. The penalty method (PM)

The penalty method (PM) transforms the constrained problem into an unconstrained problem by substituting a penalty function for the constraints. The idea is to penalize constraint violation by adding a sequence of penalty functions to the objective function in such a manner that the solutions to the resulting sequence of unconstrained problems tend to a constrained minimum (Luenberger, 1984). In this paper the most widely used exact penalty function; the absolute-value penalty function is applied. The direct set algorithm (Press et al., 1988) is used to eliminate the need to calculate the derivative of the mathematical system in finding a local optimum.

3.3. The sequential quadratic programming method (SQP)

The SQP method solves a sequence of quadratic sub-problems derived using the first and second order terms of the Taylor expansion of $f(\boldsymbol{u})$ and linear approximations of the constraints $g(z)$ (Nemhauser et al., 1989). At each iteration of the method a Quadratic Programming (QP) problem is solved using the Wolfe algorithm (Wolfe, 1959). The Hessian matrix of the Lagrangian function of the quadratic approximation is updated by a positive-definite quasi-Newton approximation using the BFGS method (MATLAB, 1992). The solution procedure initially involves the calculation of a feasible point (if one exists), and then the generation of an iterative sequence of feasible points which achieve convergence to the solution.

3.4. The sequential reduced-system programming method (SRSP)

This method is specifically designed to enable the optimization of large-scale dynamical systems. Here the system is reduced to a smaller-scale representing only the core of the original system (Janssen and Vrieze, 1995). This technique is related to the two-step algorithm designed to solve the problem of combined identification and dynamic optimization as developed by Haimes and Wismer (1972) in that we sequentially solve the dynamical system and a dynamic optimization problem for a reduced version of the system. Powells' direct set method (Press et al., 1988) is used as a local search routine.

4. Analysis

Since climate change policy has numerous objectives, any formulation of an optimization problem will necessarily neglect some specific aspects, and the problems which are examined in this study can therefore only be seen as possible illustrative examples in climate policy. The analysis concerns the investigation of three problems with two starting points. Investments are assumed to be at an equal level at each starting point, all of which are based on optimal strategies in the absence of environmental policy. We propose to cover the range of possible policy options based on IPCC scenarios (IPCC, 1991) by referring to two levels of emission reductions:

- *Business-as-Usual (BaU)*: a continuation of current trends.
- *Accelerated Policies (AP)*: a shift towards non-fossil fuels is adopted.

Furthermore we distinguish two scenarios for $\boldsymbol{u}$: BaU and AP. This leads to four start scenarios whereby the resulting temperature increase in 2100 compared to 1900 is given in Table 1. If the Business-as-Usual policy is followed for all sources it is expected that temperature will increase by some 4°C. However, if fossil CO_2 as well as other sources, are controlled according to the Accelerated Policies scenario the temperature increase will be limited to 1.3°C. As shown in Table 1, other starting points lead to the global-mean temperature increases in 2100 compared to 1900 for the various cases.

Table 1
Global-mean temperature increase x_{156} in 2100 as projected by OMEGA (in °C)

$z \setminus \boldsymbol{u}^{(5)}, \boldsymbol{u}^{(8)}$	BaU	AP
BaU	3.7	2.7
AP	2.3	1.3

The purpose of the considered optimization problems is to control z, given $\boldsymbol{u}$, to maximize the discounted sum of utility of future consumption given constraints on environmental impacts. Based on Nordhaus (1992, Nordhaus, 1993, 1994), we propose to use the following problem formulations:

(1) *Cost-benefit.*

This case is a straightforward cost-benefit optimization problem. The impacts of emission reduction costs and damage costs of a temperature increase are included in the objective function. The solution to this problem is an economically-efficient policy designed to slow climate change.

(2) *Maximum concentration of CO_2.*

In order to appraise the effects of anthropogenic emissions, we limit the increase concentration of greenhouse gases. In several studies, such targets have been used in developing response scenarios (e.g. Krause et al., 1989; IPCC, 1991). Translated to CO_2 concentration an upper limit of 400 ppmv (Krause et al., 1989) will be used.

(3) *Maximum temperature increase.*

From an environmental perspective, UNEP's Advisory Group on Greenhouse Gases (AGGG, 1990) has identified several climate targets in order to protect the structure and functions of vulnerable ecosystems and to limit risks for society. Such a target might be an absolute temperature limit of 2°C above pre-industrial level. This temperature limit can be viewed as an upper limit beyond which risks of considerable damage are expected to increase rapidly.

5. Results

5.1. Optimal responses

5.1.1. Emissions

In the case of the optimal cost-benefit formulation, the emissions of fossil CO_2 rise to a level of 16 GtC in 2100 which is about 10 GtC above the present level (Fig. 1). To meet the CO_2 concentration target, the fossil CO_2 emissions stabilize at a level of 5 GtC. The control of non-CO_2 gases does not influence the optimal policy of the first two problem formulations. However, including a temperature target in the search for suitable future greenhouse policy leads to large differences in emission reductions whether emissions of other gases are reduced or not.

In the event that emissions of greenhouse gases other than fossil CO_2 are not reduced, fossil CO_2 has to be phased out by the end of the next century in order to meet the 2°C temperature increase target. An additional reduction in emissions of other greenhouse gases results in a modest emission reduction path.

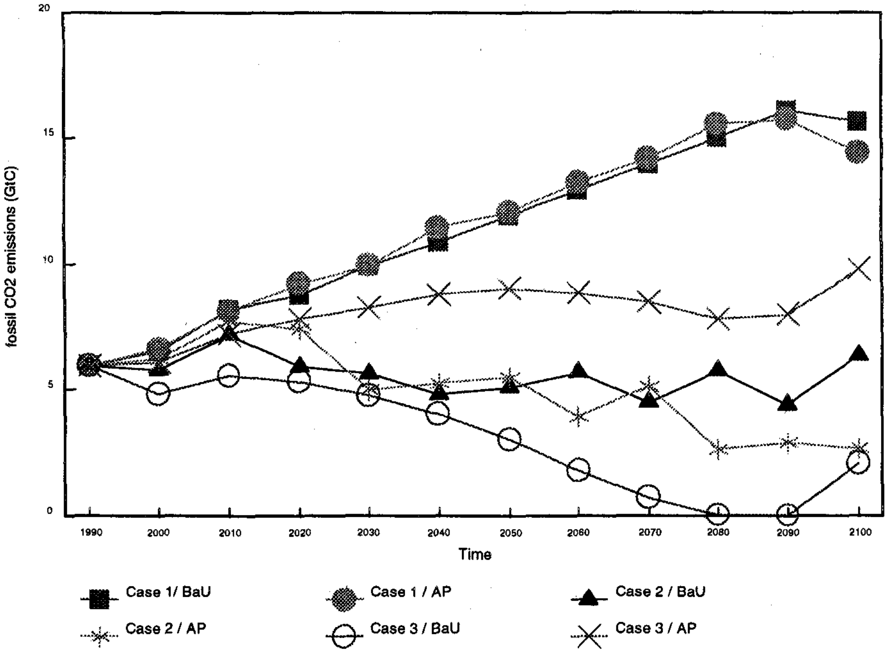

Fig. 1. Optimal emission paths of fossil CO_2 using OMEGA (in GtC).

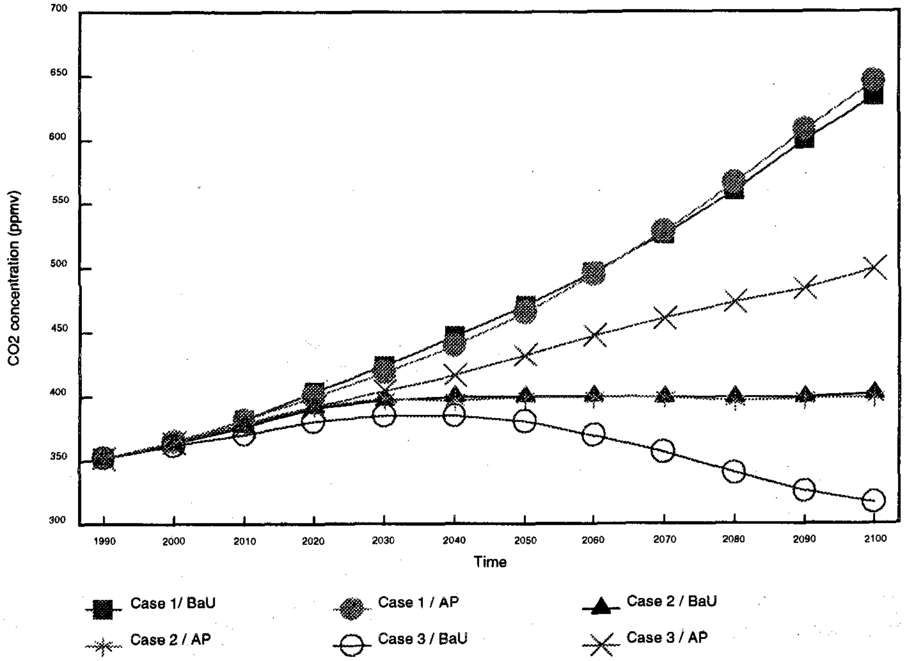

Fig. 2. CO_2 concentrations pertaining to optimal strategies according to OMEGA (in ppmv).

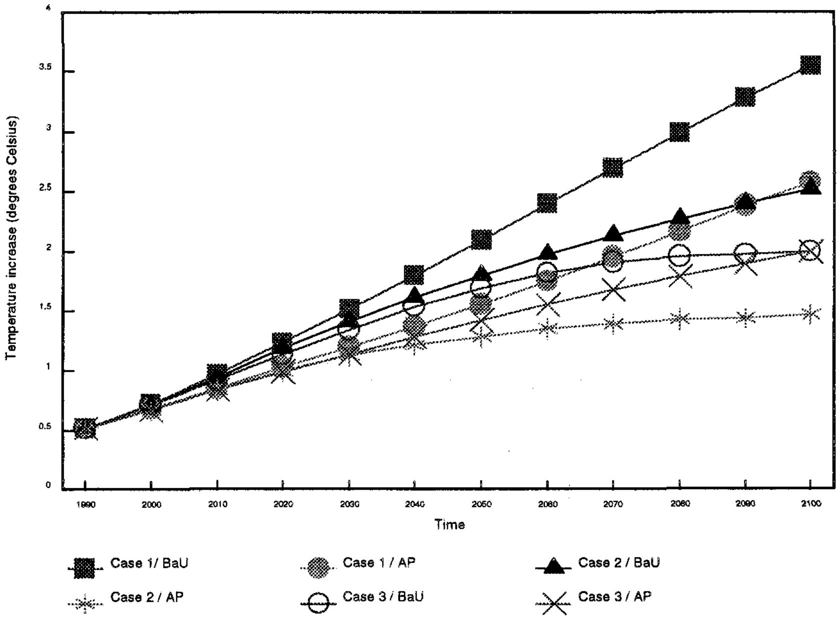

Fig. 3. Temperature increase (according to OMEGA, in °C relative to 1900).

5.1.2. *Concentration of CO_2*

If environmental constraints are not explicitly taken into account (Case 1), CO_2 concentration will reach a level of 630 ppmv by 2100 (Fig. 2). A phase out of fossil CO_2 emissions (case 3 including BaU scenarios for non-fossil CO_2 emissions) is the only strategy which leads to a decrease in the CO_2 concentration.

5.1.3. *Temperature increase*

The extra temperature increase associated with the case not all greenhouse gases are controlled leads to an additional 1°C temperature increase. Therefore a cost-benefit policy leads to a temperature increase of between 2.6 and 3.5°C, while meeting the concentration target results in a range of 1.5 to 2.5°C (Fig. 3). The reader will note that the temperature increases are significant in the next century and in all cases may lead to serious impacts on economic, agricultural and social structures and practices.

5.1.4. *Output*

The differences in economic output among the various cases are relatively small throughout the next century (Fig. 4). Output will, at most (in the case of a phase-out of fossil CO_2 emissions) lead to a 12% lower output than in the optimal cost-benefit case. Note that the economic output here is "green" in that it takes account of environmental impacts of economic activities in terms of market damage.

Table 2
Performance criteria

Accuracy	
Objective:	percentage of difference with the best solution found.
Constraints:	sum of violation of constraints ($\Sigma g(u)$).
Efficiency	
Execution time (seconds):	cpu time on Silicon Graphics (Indy) workstation.
Function evaluations:	number of model runs.
Reliability	
Failures (%):	percentage of runs where no feasible solution was found.

5.2. *Performance results*

The performance of the optimization approaches is evaluated for the solutions of the three test problems, starting the search at two starting points. In order to quantify the performance, criteria are distinguished based on Hock and Schittkowski (1983), which is shown in Table 2.

Table 3
Performance of the 3 optimization approaches on the test problems. (Only feasible solutions are used to determine the accuracy score). The function evaluations of the reduced system are given between parentheses (SRSP)

	Accuracy		Efficiency		Reliability
	Objective	Constraints	Run time	f.e.	% failures
Penalty Method	0.096%	0.000018	15443.8	3907.4	0%
SQP	0.821%	0.000000	8897.4	2096.7	58%
SRSP	0.023%	0.003823	3742.1	16.8 (37148.8)	25%

The resulting average scores are shown in Table 3.

The penalty method which uses the original model, thus is able to cope with all of the dynamics inherent in the problem, was found to be a reliable and accurate but time-consuming approach for solving the large scale non-linear optimization problem. The SRSP method is as accurate and far more efficient as the penalty method in most of the cases, although it did not lead always to convergence for one problem (case 3: maximum temperature increase). Janssen and Vrieze (1995) showed that SRSP leads to convergence for a large class of problems, however for some cases the reduced system is not able to capture the dynamics of the original system such that convergence can be guarantied. Sequential Quadratic Programming is unreliable for the kind of (constraint) problems as considered in this study as a result of the high degree of non-linearity which is not approximated within a quadratic framework.

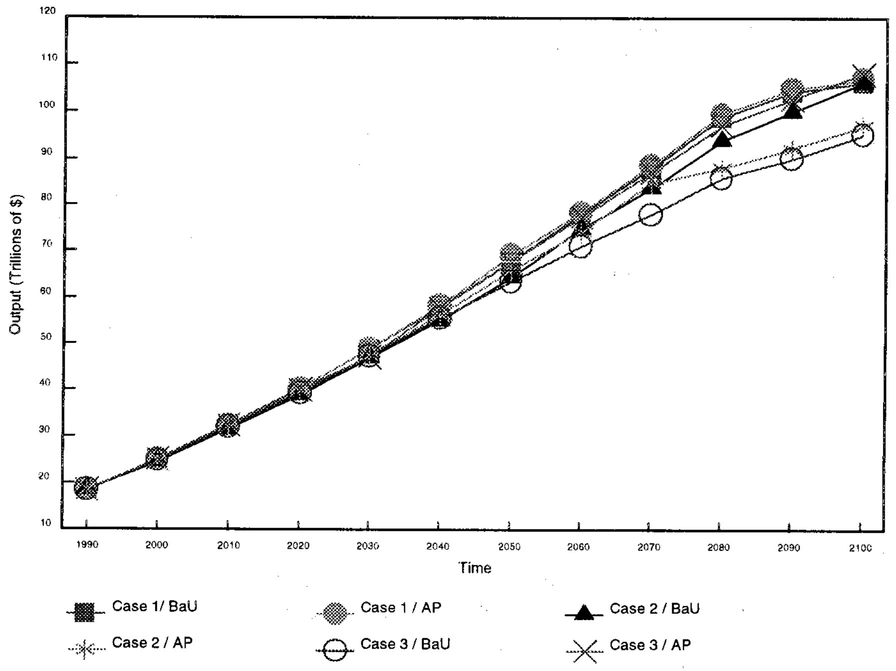

Fig. 4. Economic output (according to OMEGA).

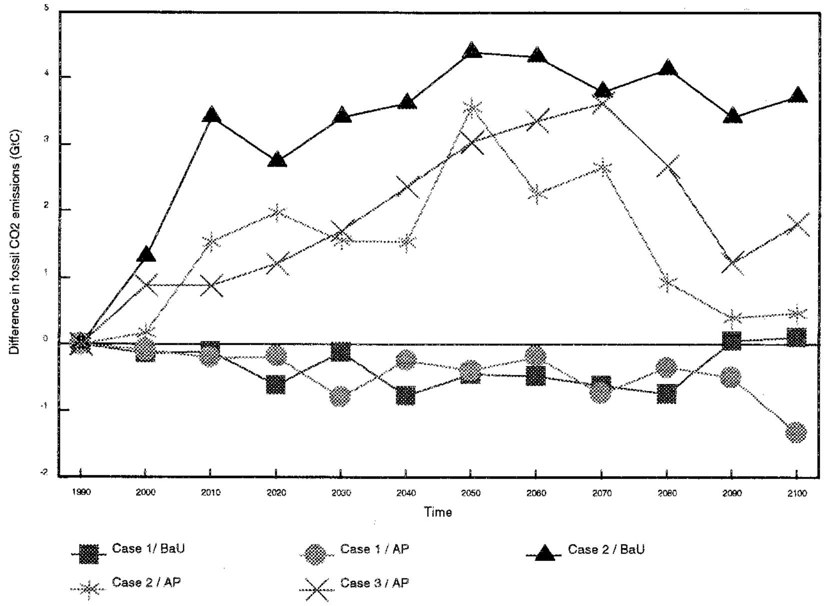

Fig. 5. Difference in optimal emission paths between OMEGA and DICE (in GtC).

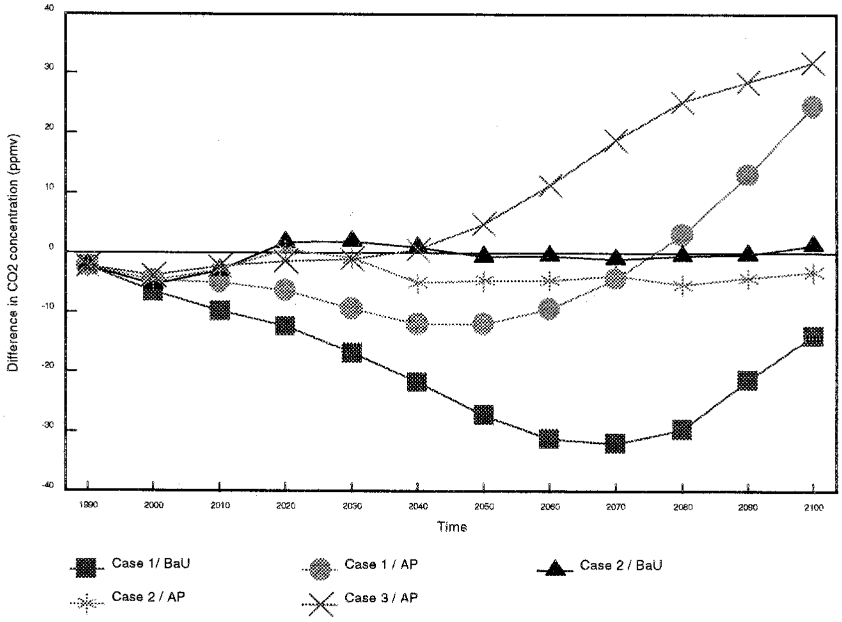

Fig. 6. Difference in CO_2 concentration between OMEGA and DICE (in ppmv).

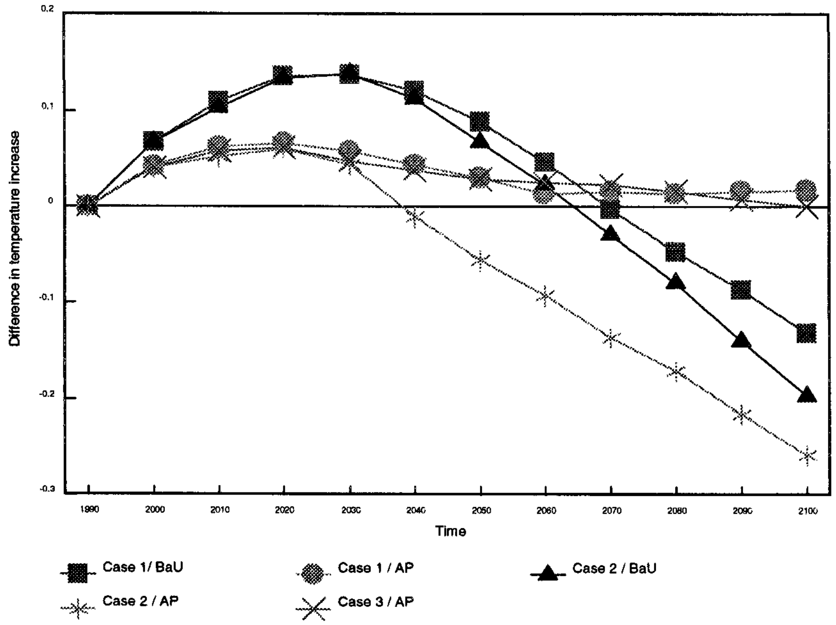

Fig. 7. Difference in temperature increase between OMEGA and DICE (in °C).

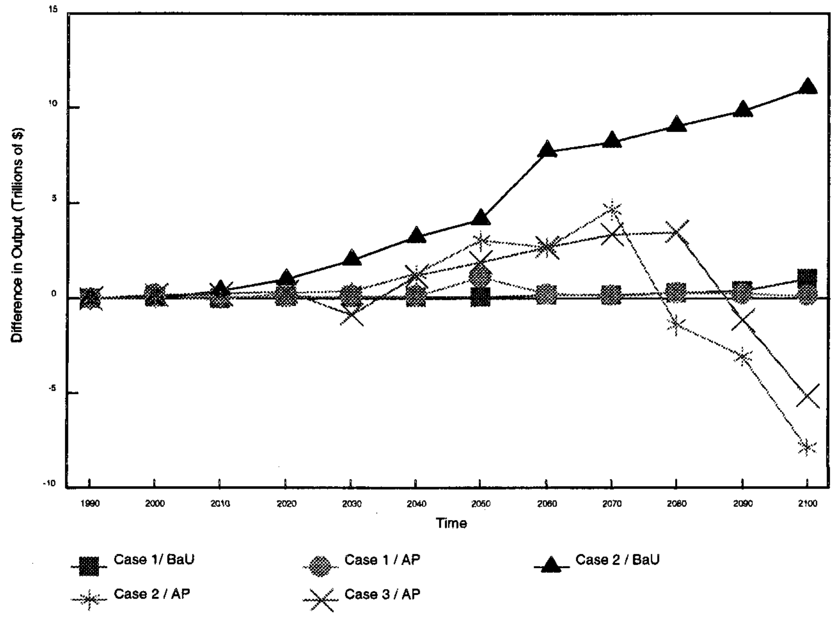

Fig. 8. Difference in economic output between OMEGA and DICE.

Therefore, quadratic subproblems do not help the search for the optimal solution but leads to the rapid exhaustion of the search algorithm.

5.3. Comparing results of OMEGA with DICE

The optimization problem was rendered a large-scale problem by replacing the small climate model of DICE by the 155 dimensional mathematical system which features in the IMAGE 1.0 model. In this section we will discuss the differences which occur as a result of using the original DICE model and the derived OMEGA model.

In order to compare the results we propose to start by referring to identical "business-as-usual" results for both models, to which end we slightly adapt the DICE model. Originally, the temperature increase in 2100 for the DICE model was 3.3°C (Nordhaus, 1992, 1993, 1994). If we use the business-as-usual scenario borrowed from IMAGE for the other trace gases ($O(t)$, Eq. (7), and if we only wish to control fossil CO_2 emissions, and furthermore assume exogenous levels of CFC emissions and CO_2 emissions due to land use changes, the temperature increase in 2100 is calculated as 3.8°C.

The cost-benefit case solution does not significantly differ from the solution produced by OMEGA (Figs. 5–8). OMEGA results support the case for a somewhat enhanced emission reduction strategy compared with DICE results. However, in order to meet the concentration target, emissions have to be stabilized at a level of 1 GtC, which is 80% lower than the level envisaged by OMEGA. The temperature target can only be met if other greenhouse gases are also controlled.

Table 4
Emission budgets of fossil CO_2 for the period of 1990 to 2100 in GtC, for BaU as well as for AP policy for other greenhouse gas emissions

Case\Model	DICE		OMEGA	
	AP	BaU	AP	BaU
Cost benefit	1300	1285	1280	1245
Concentration target	239	239	598	612
Temperature target	529	–	888	338

The differences in emission budgets for the next century can be read off from Table 4. The inclusion of environmental constraints leads to larger reductions in emissions according to DICE as opposed to OMEGA. The climate model incorporated in DICE and especially as expressed in Eq. (7) (the atmospheric concentration of CO_2) is based on historical trends (see also Price, 1995). The DICE model is therefore only able to estimate future concentrations by extrapolating historical behaviour. If, however, in the next century, environmental policies were to be implemented which would result in a transformation of economic activities resulting in different patterns of fossil CO_2 emissions, the DICE model would no longer be able to capture the consequences of such changes. OMEGA is designed to describe the climate processes insofar as they are understood by natural scientists, and is therefore able to capture a wider range of future scenarios.

The results show that within the OMEGA framework, limiting temperature change would be relatively less expensive than within the DICE model, which results in a lower emission path in the case of a cost-benefit analysis. Consequently, meeting environmental constraints is far more expensive if we use DICE compared with the results of OMEGA. To estimate the impact of different approaches in terms of cost, the present value of future consumption is calculated. The loss of future consumption is assumed to be the difference between the optimal policy without additional environmental constraints (assuming BaU scenario emissions for non-fossil

Table 5
Loss of discounted value of consumption compared to the reference case (cost-benefit assuming AP scenarios for non-fossil CO_2 emissions) (in trillions of 1989 US $). The loss of consumption as percentage of the discounted value of consumption of the reference case is given between parentheses

Case\Model	DICE		OMEGA	
	AP	BaU	AP	BaU
Cost benefit	– 2.6 (– 0.2%)	0.0 (0.0%)	– 2.8 (– 0.3%)	0.0 (0.0%)
Concentration target	41.2 (3.8%)	66.4 (6.2%)	19.0 (1.8%)	20.7 (1.9%)
Temperature target	12.8 (1.2%)	–	1.6 (0.1%)	43.6 (4.1%)

CO_2 emissions) and considered policy (Table 5). The loss of the value of consumption goes up from 1% to 6% in the event of additional environmental constraints are included. This exercise shows that the results are highly dependent on the manner in which the climate dynamics are modelled and underlines the importance of using models which rely on the best available scientific knowledge. Compared to Nordhaus (1992, Nordhaus, 1993, 1994), the loss of consumption is in all cases significantly higher because in OMEGA only fossil CO_2 is controlled, neglecting the relatively cost-efficient reduction in CFCs and CO_2 emission due to land use changes.

6. Conclusions

OMEGA combines the strongest component of two integrated assessment models: the economics of DICE and the climate dynamics of IMAGE. The resulting optimization model allows us to scan the range of possible response policies in order to meet targets for economic development and environmental protection. The integration generates a large-scale non-linear optimization problem which is solved by employing different approaches with different degrees of success.

Cost-benefit analysis leads to an enhanced effort in reducing emissions if OMEGA is used compared with DICE. However, in the event of severe constraints on concentration levels or temperature change, far less emission reductions have to be made if we use OMEGA instead of DICE. These differences could be explained by the different descriptions of the dynamics of the climate system.

In short, this paper shows that it is possible to incorporate more sophisticated models of the climate system in optimization studies of the greenhouse effect and that such an integration has important impact on the derived solutions.

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