AN APPLICATION OF OPTIMIZATION TO THE PROBLEM OF CLIMATE CHANGE

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ABSTRACT

The objective of this paper is to demonstrate a methodology whereby reductions of greenhouse gas emissions can be allocated on a regional level with minimal deviation from the "business as usual emission scenario". The methodology developed employs a two stage optimization process utilizing techniques of mathematical programming. The stage one process solves a *world emission reduction problem* producing an optimal emission reduction strategy for the world by maximizing an economic utility function. Stage two addresses a *regional emission reduction allocation problem* via the solution of an auxiliary optimization problem minimizing disruption from the above business as usual emission strategies. Our analysis demonstrates that optimal CO_2 emission reduction strategies are very sensitive to the targets placed on CO_2 concentrations, in every region of the world. It is hoped that the optimization analysis will help decision-makers narrow their debate to realistic environmental targets.

1 INTRODUCTION

At first sight, it might appear that global environmental problems such as the greenhouse effect have no relation to the subject of mathematical programming. However, the widely anticipated international agreements on reductions of emissions of greenhouse gases[1] will involve complex tradeoffs among signatories of such agreements. These tradeoffs will, for the most part, be between industrialized countries – countries that contribute most to the current greenhouse problem – and developing countries

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[1] For example agreements made at the Montreal and Rio conferences

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aspiring to the standard of living of the industrialized world which in turn seems to demand commensurate levels of energy generation.

Not all countries have contributed equally nor will they be affected equally by the anticipated climatic changes caused by the increasing concentrations of greenhouse gases in the atmosphere. Furthermore, not all countries will be equally able to adapt to climatic changes if and when they occur, nor will they be able to absorb the economic costs associated with reducing greenhouse gas emissions. Consequently, not all countries will want to reduce atmospheric concentrations of greenhouse gases by way of internationally agreed upon emission reduction strategies. For instance, some countries may be worse off by participating in such agreements and will therefore attempt not to cooperate. Countries that are better off under such agreements may also prefer not to participate as it may be potentially more profitable for them to behave as a *free rider*[2]. Nonetheless, international agreements on the so-called *carbon taxes* form the core of the most frequently discussed response strategies to the threat of the enhanced greenhouse effect.

In this paper we adopt the point of view that these agreements can, and perhaps, should, be regarded as a two-stage optimization problem of the generic form:

Stage 1. Find $\mathbf{U}^0$, a world CO_2 emission strategy, that

$$\begin{array}{llll} & \text{maximizes} & C(\mathbf{U}) & \\ \text{s.t.} & & & \\ \text{(i)} & e_i(\mathbf{U}) = 0 & i \in I & \text{– economic dynamics} \\ \text{(ii)} & p_j(\mathbf{U}) = 0 & j \in J & \text{– climate dynamics} \\ \text{(iii)} & g_k(\mathbf{U}) = 0 & k \in K & \text{– environmental targets constraints} \end{array}$$

where $C(\mathbf{U})$ is a world economic utility function. Once an optimal $\mathbf{U}^0$ is obtained from above:

Stage 2. Find u_r, a regional CO_2 emission strategy (for each world region r) that

$$\begin{array}{lll} & \text{minimizes } D(u_1, u_2, \ldots, u_\rho) & \\ \text{s.t.} & & \\ \text{(i)} & \sum_r u_r = \mathbf{U}^0 & \text{– consistency with international agreements} \\ \text{(ii)} & m_r u_r^A \le u_r \le M_r u_r^A, & \text{– practical constraints on emission reductions} \\ & \text{where } r = 1, 2, \ldots, \rho. & \end{array}$$

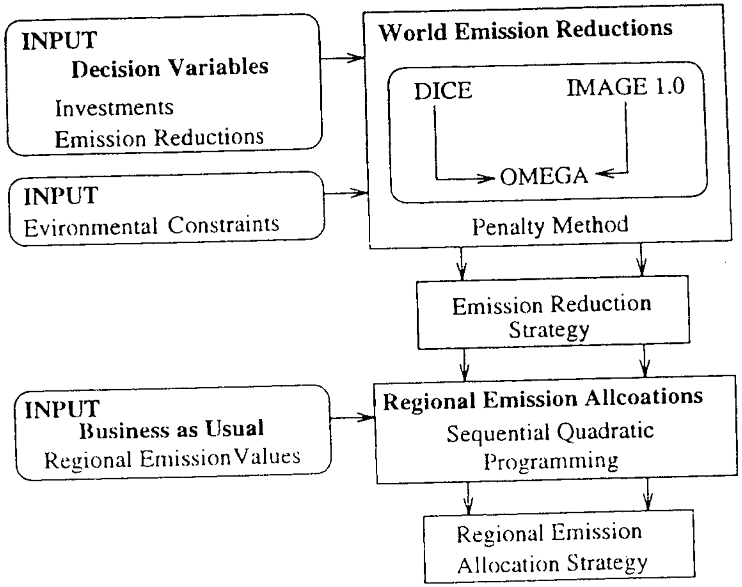

Figure 1 The OMEGA+ Environmental Optimizer

where D is a function that captures the level of "*disruption*" caused by deviations from the so called *business as usual*[3] strategy $(u_1^A, u_2^A, \ldots, u_\rho^A)$. The constraints (ii) ensure that no world region is required to reduce its CO_2 emissions by an unrealistically large, or small, amount.

Using a two stage environmental optimizer, OMEGA+ (Optimization Model for Economic and Greenhouse Assessment), this paper demonstrates a methodology for the creation of possible regional emission reduction strategies whereby agreed upon world emission reductions can be achieved with minimum deviation from the IPCC[4] (Intergovernmental Panel on Climate Change) business as usual strategy (IPCC, 1991, 1992). OMEGA+ incorporates a coupled economic/global climate change model with a regionalized global optimization emission allocation procedure as seen in Figure 1.

Stage one of the environmental optimizer constitutes solving a *world emission reduction problem* (based on Janssen 1995), maximizing an economic utility function, for

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[2] A free rider is defined to be someone that benefits because of certain actions without assuming any of the responsibility or costs involved in achieving such actions.

[3] The business as usual scenario assumes a continuation of economic growth without restriction from environmental constraints.

[4] United Nations panel setup by the World Meteorological Organization (WMO) and the United Nations Environmental Programme (UNEP) to study impacts of climate change (See IPCC, 1991, 1992)

the entire world, subject to climate dynamics, economic dynamics and environmental target constraints. Stage one yields an "*optimal*" world emission reduction strategy, which we denote by $U^0(\cdot)$.

The solution to stage two ***regional emission reduction allocation problem*** is based on an auxiliary optimization problem in the form of minimizing disruption from business as usual strategy (Section 2) subject to being consistent with $U^0(\cdot)$, and regional practical constraints (Figure 2).

Remark: In principle, it should be possible to combine the two stages into a single optimization problem in the regional decision variables. Because of several technical reasons (including the availability of data), this would be more difficult. In addition, the two stage approach seems to correspond more closely to the manner in which carbon taxes are likely to be brought about. Namely, internationally negotiated emission reduction targets will induce optimization problems at lower levels.

2 EMISSION SCENARIOS

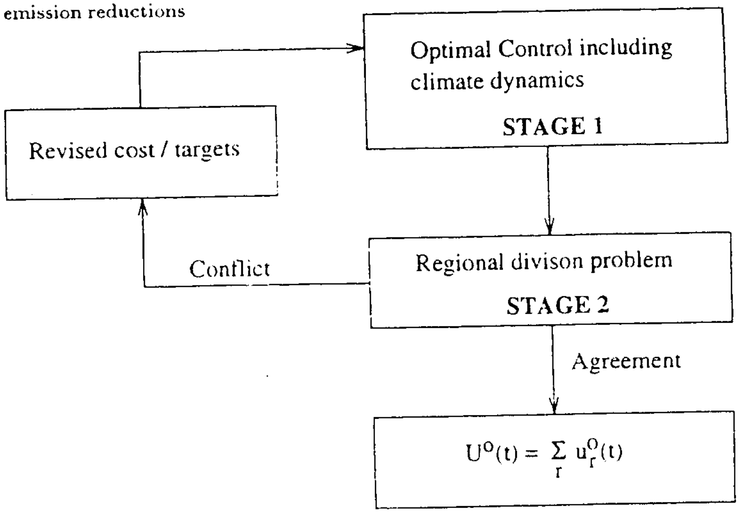
In the greenhouse modelling community emission scenarios for greenhouse gases such as carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O) and the chlorofluorocarbons (CFCs) form the core of environmental debate. In 1990, the IPCC put forward a set of four environmental emission scenarios based on consistent assumptions for each of the greenhouse trace gases. The scenarios were meant to encompass possible global, socio-economic development factors in order to illustrate the impact of different future pathways on the greenhouse effect. The scenarios put forward were the ***business as usual*** (or unrestricted trends), ***reduced trends, changed trends*** and ***forced trends*** (or accelerated policies scenario), details of which are described in IPCC, (1991, 1992).

However scenarios are not firm predictions of the future and should not be used as such. This becomes increasingly true as the time horizon increases, because the basis for the underlying assumptions becomes increasingly speculative. There are considerable uncertainties surrounding the evolution of the types and levels of human activities (including economic growth and structure), technological advances, and human responses to possible environmental, economic and institutional constraints.

3 BACKGROUND

There are several recent *Optimization Models* investigating climate change and economic activity (Nordhaus, 1992, 1993, 1994 and Tahvonen et. al., 1993). These models although comprising an extensive economic component, contain extremely simplified versions of the climate system thereby giving only a limited representation of the underlying system processes and climate dynamics (Janssen et. al., 1995).

Figure 2 Iterative process for arriving at mutually acceptable targets and emission reductions

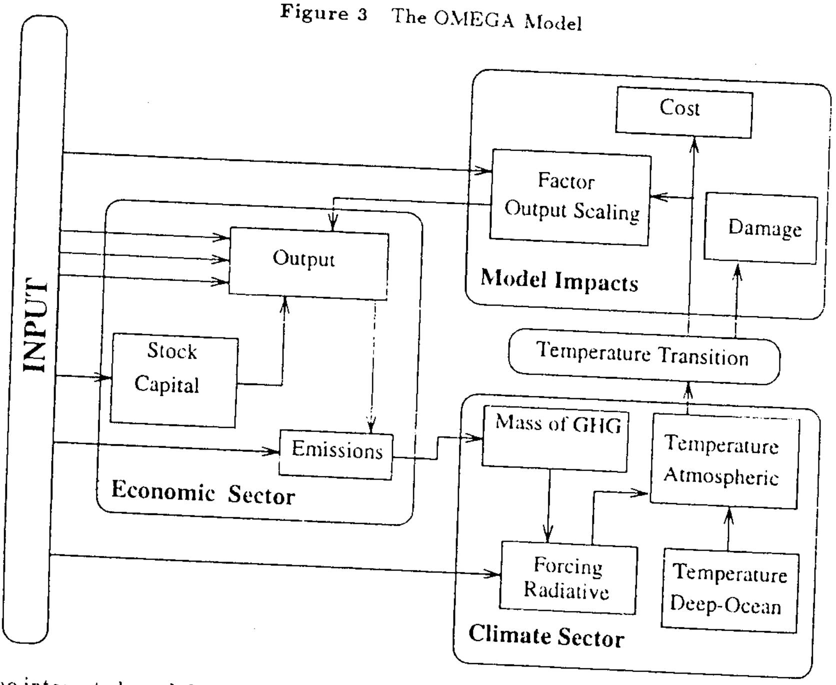


On the other hand, there are also *Integrated Assessment Models* which, include a more comprehensive account of the dynamics associated with the climate system. However, these integrated assessment models tend to be somewhat larger than the environmental optimization models of Nordhaus and Tahvonen, causing them to have considerably longer execution times. Furthermore, some of these integrated assessment models contain an inadequate economic component and hence statements about the economic significance obtained from these models are difficult to make.

The OMEGA model (Janssen, 1995) within the OMEGA+ environmental optimizer combines an economic optimization model DICE (Dynamic Integrated model for Climate and Economy, Nordhaus 1992, 1993 and 1994) with a climate integrated assessment model IMAGE 1.0 (Integrated Model to Assess the Greenhouse Effect, Rotmans, 1990). The model permits the strengths of each of the combined DICE and IMAGE 1.0 models to overcome the weaknesses of the other.

DICE, primarily an economic optimization model with highly simplified climate dynamics (Nordhaus, 1992, 1993 and 1994), calculates optimal trajectories for both *capital accumulation* and *greenhouse gas emission reductions* by maximizing a discounted value of "*utility*" or satisfaction from consumption[5] subject to economic and geophysical constraints. The model is essentially an extension to the Ramsey model (Ramsey, 1928) of optimal economic growth to environmental policy.

Figure 3 The OMEGA Model



The integrated modular system of IMAGE 1.0 incorporates relatively simple modules of the main components of the global greenhouse effect (Rotmans, 1990). The model attempts to describe the major cause and effect relationships with respect to climate change. A mathematical system extracting the core of IMAGE 1.0 as a system of 155 differential equations has been developed in (Braddock et. al., 1994) and it is this system that is used in the OMEGA model. Figure 3 represents the logical structure of the OMEGA model.

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[5] Here consumption is a broad concept including not only traditional market purchases of goods and services but also non-market items such as leisure, culture and amenities.

4 STAGE ONE: WORLD EMISSION REDUCTION PROBLEM

The time period under consideration is 1990 – 2100. Following Nordhaus (1994) we assume that the levels of decision variables change linearly over fixed time intervals of ten years. That is, we first impose a coarse time discretization of eleven intervals $[t_0, t_1], [t_1, t_2], \dots, [t_{10}, t_{11}]$, where $t_0 = 1990, \dots, t_{10} = 2090$, and $t_{11} = 2100$.

In stage one we pose the following world emission reduction optimization problem with decision variables $\mathbf{z}(t) = (z_1(t), z_2(t))$, where $z_1(t)$ defines the rate of CO_2 emission reductions at time t and $z_2(t)$ defines rate of investment in tangible capital at time t.

$$\max f(\mathbf{z}) \tag{1.1}$$

s.t.

$$\Phi_t(\mathbf{z}) \le \rho CO_2; \qquad t = 1990, 2010, \dots 2100$$
$$0 \le \mathbf{z} \le 1$$

where $f(\mathbf{z})$ defines a world economic utility function (Appendix B). $\Phi_t(\mathbf{z})$ define environmental target constraints at which we fix the limits on atmospheric CO_2 concentration, ρCO_2. In the above,

$$\mathbf{z} = \{\mathbf{z}(t) | t = 1990, 2000, \dots, 2100\}.$$

Stage one is solved utilizing the penalty method and the Powell direction set method (Press et. al., 1988 and Brent, 1973). The penalty method transforms the constrained problem to an unconstrained one by substituting a penalty function for the constraints. The idea is to penalize constraint violations by adding a penalty function to the objective in such a way that the solutions to the resulting sequence of unconstrained problems tend to a constrained maximum (Luenberger, 1984). For instance, violations of the constraints on CO_2 concentrations are penalized by a function of the form:

$$\phi \sum_{k=1}^{T} \max \{0, (\Phi_k(\mathbf{z}) - \rho CO_2)\}. \tag{1.2}$$

5 STAGE TWO: REGIONAL EMISSION REDUCTION ALLOCATION PROBLEM

Regional allocations will be made on a four region basis: OECD, former USSR and Eastern Europe, China and centrally planned Asia, and the rest of the world (RoW). The 145 countries embodied in these regions are those with a population of at least one million or a GDP of at least US \$1 bn (Economist, 1990). A list of countries contained within each region is given in Appendix C. We base our solution to the allocation problem on the solution of an auxiliary optimization problem in the form of minimizing disruption due to emission reductions subject to these emission reductions being consistent with the emission reduction strategy $U^0(\cdot)$ found in stage one, and regional practical constraints. The results from the stage one problem need to be decomposed into regional values $u_r(t)$. Initially we propose the following stage two optimization problem which will yield a global minimum:

$$\min \sum_t \sum_r \omega_r (u_r^A(t) - u_r(t))^2$$

s.t.

$$\begin{aligned} &\text{(i)} \sum_r u_r = U^0 && \forall t \\ &\text{(ii)}\ m_r u_r^A \le u_r \le M_r u_r^A && \forall r, t \end{aligned} \tag{1.3}$$

where (ii) consists of "practical" constraints such as: permitting a region to emit at least $m_r\%$, but no more than $M_r\%$[6] of its business as usual scenario emissions. We define ω_r to be the weight associated with region r, and $u_r^A(t)$ to be the regional emissions due to the IPCC business as usual scenario in year t. Note that (1.3) will be a convex program in the decision variables $u_r(t)$ and hence a global minimum should be computable. The interpretation of (1.3) is that it will provide "*minimum disturbance*" emissions for each region, subject to the total being the "*optimal*" emissions from stage one. Weights on time periods are also possible. Because of the availability of data the decision times in the stage two problem will be taken at ten year intervals.

If $u_r(t)$'s are supplied as solutions of (1.3) and if there is an international agreement on the decomposition then for each $t = 1990, 2000, \ldots, 2100$

$$U^0(t) = \sum_r u_r(t) \tag{1.4}$$

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[6] It is possibly that M_r could be more than 100% and thus allowing region r's CO_2 emissions to exceed that of the business as usual scenario.

where r denotes the regions. If an agreement is not met, targets will be revised (this may include revising the values for $m_r(t)$ and $M_r(t)$) and the optimizer re-executed. The process is thus an iterative one (Figure 2).

The constrained minimization of stage two was performed using the constrained optimization toolbox within MATLAB 4.2 (MATLAB, 1992). The solution methodology is based on the solution of the Kuhn-Tucker (KT) equations, which are necessary and sufficient conditions for optimality for a convex program. By using a constrained quasi-Newton method we guarantee superlinear convergence by accumulating second order information regarding the KT equations. This information is gathered by way of a quasi-Newton updating procedure utilizing the BFGS method (Fletcher, 1980 and MATLAB, 1992). Consequently, the method applied is referred to as *Sequential Quadratic Programming* (SQP) (Fletcher, 1980 and Gill et. al., 1981). For a comprehensive overview of global optimization see Horst and Pardalos, (1995).

6 OMEGA FORMULATION

A major difficulty to be overcome is the fact that the complexity of the underlying climate and economic dynamics is such that it is impractical to derive the constraint functions $\Phi_t(z)$ explicitly. However, they can be evaluated iteratively using the difference equations of IMAGE 1.0 and DICE (see below). The continuous time version of the IMAGE 1.0 dynamical system (Appendix A) takes the form:

$$\dot{x}(t) = F(x(t)) + u(t) \quad x(1990) = x_0 \tag{1.5}$$

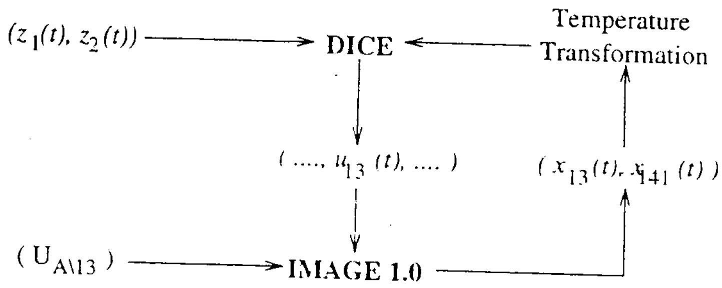
and can be approximated, via time discretization, by:

$$x(t+1) - x(t) = F(x(t)) + u(t), \tag{1.6}$$

where $u(t)$ represents the forcing or human interference term included in the climate system process. Note that a time step of ten years is far too large to justify the discretization of the dynamical system in (1.5). Consequently we further discretize each of the ten year intervals by setting:

$$\begin{aligned} \tau_{k,j} &= t_k + j; && j = 0, 1, \ldots, 9 \\ \tau_{k,9} &= t_{k+1}; && k = 0, 1, \ldots, 10. \end{aligned}$$

Figure 4 Coupling of IMAGE 1.0 and DICE



The economic component from DICE (Appendix B) which we couple with (1.6) takes the form:

$$\mathbf{e}(t+1) = \mathbf{E}(\mathbf{e}(t), z_2(t)) \tag{1.7}$$

where $\mathbf{e}(t)$ is a vector representing the economic state variables of the model and $z_2(t)$ represents rate of investment in tangible capital at time t (see Appendix 2). Coupling IMAGE 1.0 and DICE are the variables *temperature of the mixed ocean layer* $x_{141}(t)$ and the *atmospheric carbon dioxide concentration* $x_{13}(t)$ (Figure 4)[7]. The coupling is somewhat more complex as the economic component of DICE requires the surface temperature above land as input, whereas IMAGE 1.0 only yields the *temperature of the mixed ocean layer* as output. Consequently, OMEGA introduces additional terms into the model to provide a transformation between these variables, see (B.12) in Appendix B and Janssen (1995).

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[7] $U_{A\backslash 13}$ indicates that the climate assessment component of OMEGA takes as input the IPCC business as usual scenario, denoted by A, except for the 13th component which is supplied by the economic component.

Hence starting at $t = 1990$ the environmental constraint

$$x_{13}(1990) \leq \rho\mathrm{CO}_2 \tag{1.8}$$

can be expressed as

$$\Phi_{1990}(z_1(0), z_2(0)) = 351 \tag{1.9}$$

since CO_2 concentration in 1990 is known to be approximately 351 ppm (parts per million). Now by iterating (1.6) and (1.7), at an intermediate time such as $t = 1996$, the environmental contraint in (8) can be expressed as

$$\Phi_{1996}(z_1(0), \ldots, z_1(5), z_2(0), \ldots, z_2(5)) \leq \rho\mathrm{CO}_2, \tag{1.10}$$

and so on for higher values of t.

Since (1.6) involves a system of 155 nonlinear equations it should be clear that it would be pointless to try to derive an analytical form of the constraints $\Phi_t(\mathrm{z}) \leq \rho\mathrm{CO}_2$ for large values of t. However, in a numerical implementation of the Powell direction set method (Press et. al., 1988), only function evaluations are needed. Implicitly, at each function evaluation the dynamical system (1.5) is solved via a fourth order Runga-Kutta-Fehlberg error control algorithm (Burden and Faires, 1989). The error control theory associated with this algorithm requires roughly double the number of function evaluations per step as methods without the error control. This was considered acceptable due to the complexity of the model.

7 RESULTS

7.1 Stage One

The stage one optimization problem was solved using the business as usual scenario as a benchmark and the environmental target constraint given in (1.1) set to an atmospheric CO_2 concentrations (ρCO_2) of 400 and 500 ppm[8] (part per million),

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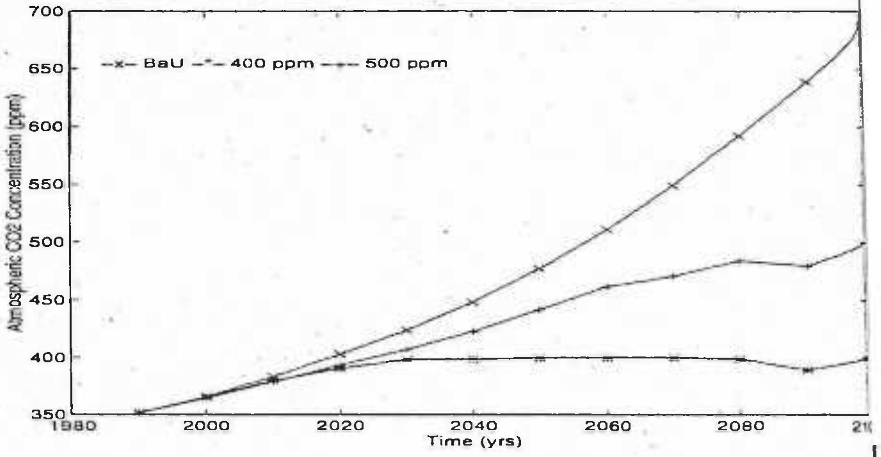
[8] Under the business as usual scenario the concentration of atmospheric CO_2 is estimated to reach approximately 680 ppm by the year 2100.

Figure 5 Atmospheric CO_2 Concentration Environmental Targets



respectively. Of course, any ρCO_2 level can be applied, however, 400 and 500 ppm were considered as targets for the more environmentally conscious IPCC emission scenarios. For each of these CO_2 concentrations, stage one finds a world emission reduction strategies $U^0(t)$ satisfying the restriction on CO_2 while also maximizing economic utility.

Levels of atmospheric CO_2 concentration for each of the environmental targets are shown in Figure 5. Figure 6 gives levels of world CO_2 emissions that must not be exceeded in order to achieve the environmental target on ρCO_2. Notice that in Figure 6 the ρCO_2 decreases slightly from the year 2080 and then increases from 2090 to the year 2100. This fluctuation is understandable in that CO_2 concentration levels after 2100 are not considered at all and thus can potentially rise to values well in excess of the environmental target. Consequently, to maximize economic utility during this period the optimization process increases CO_2 emissions sufficiently so that the environmental target constraint is still satisfied at 2100, however, it may not be satisfied immediately after 2100, see also Table 2.

Such an anomaly is inevitable when optimizing over a finite time horizon. The optimization problem does not know that the world does not end in the year 2100. One way to ensure the optimization procedure produces environmentally realistic emission strategies up to the year 2100 that will not cause excessive rises to occur immediately following 2100 is to increase the time frame of the optimization to, say, 2110 or 2120. Hence, when considering environmental policy issues from the optimization presented here one should only use values up to the year 2080. Table 1 gives the stage one optimization performance figures for each of the environmental targets.

Figure 6 CO_2 Emission Levels

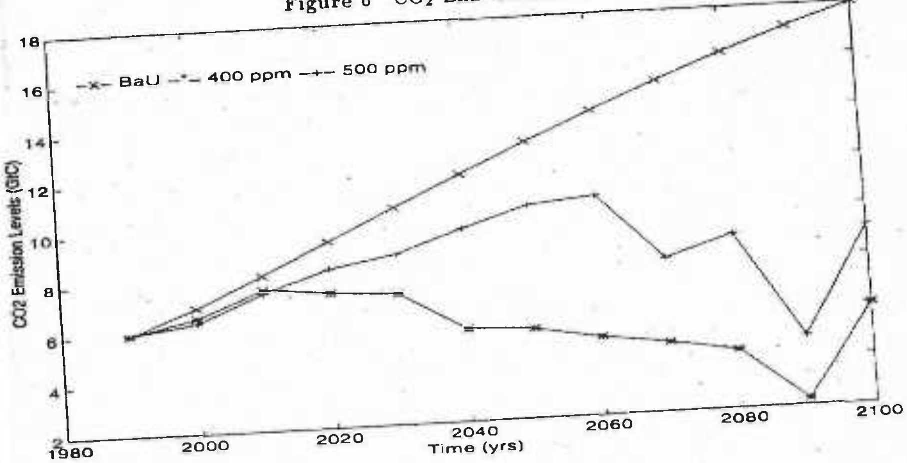


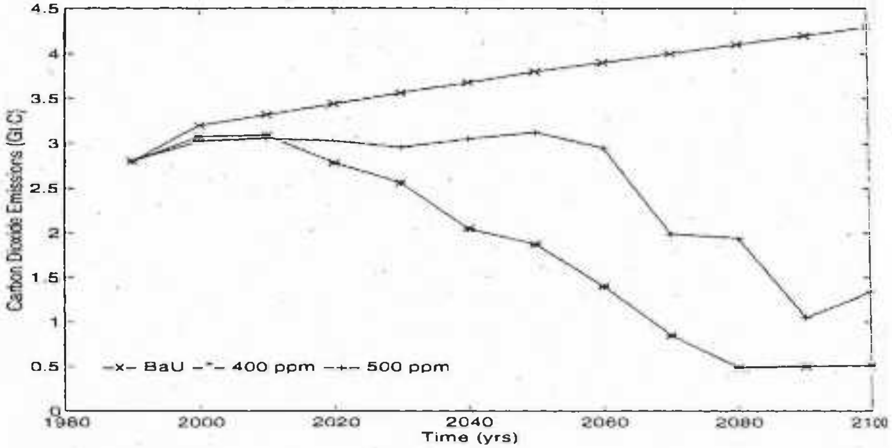
Table 1 Performance Summary

Description	$\rho CO_2 = 400$ ppm	$\rho CO_2 = 400$ ppm
Objective value (at termination)	335625.17286	337089.32454

Table 2 Decision Variable

	Rate of Emission Reductions ($z_1(t)$)		Rate of Investment ($z_2(t)$)	
	ρCO_2		ρCO_2	
Time (yrs)	400 ppm	500 ppm	400 ppm	500 ppm
1990 – 2000	0.082766	0.117936	0.216835	0.211296
2000 – 2010	0.213098	0.147334	0.208027	0.203764
2010 – 2020	0.341076	0.166672	0.222628	0.222255
2020 – 2030	0.347524	0.289391	0.205901	0.205571
2030 – 2040	0.793267	0.086695	0.202218	0.201942
2040 – 2050	0.755135	0.337034	0.206996	0.203579
2050 – 2060	0.825315	0.206062	0.201339	0.200564
2060 – 2070	0.621097	0.586455	0.198084	0.194093
2070 – 2080	0.756344	0.424072	0.213474	0.209268
2080 – 2090	0.873571	0.748743	0.189750	0.185899
2090 – 2100	0.559499	0.340173	0.000249	0.000097

Figure 7 OECD (Organization for Economic Co-operation and Development)



7.2 Stage Two

The environmental emission strategies obtained in stage one provide valuable information for reducing world atmospheric CO_2 concentrations, however, even if the entire world cooperates in reducing CO_2 emissions it is unclear by what degree each world region or country must decrease their emissions to achieve the target. Stage two attempts to provide a mechanism for optimal allocation of the maximum allowable CO_2 emission levels to achieve the stage one environmental constraint on the four world regions (Appendix C). Again using the individual regional business as usual scenarios we can find regional emission levels to satisfy the environmental target while also minimizing the disruption from these benchmark scenarios. The fluctuations around the year 2090 are also visible in stage two, since stage two uses stage one CO_2 emission levels.

For the above experiments the upper limit on the practical constraint (ii) of the formulation in (1.3) was set at $M_r = 1$, for the business as usual (BaU) scenario. Thus no region was allowed to emit levels of CO_2 above their business as usual scenario. The lower limit m_r was set to the highest value below M_r that provided a feasible solution. Consequently, the solution will provide the minimum disruption from the business as usual scenario for each region and satisfying the environmental target conditions. The regions were all given equal importance, thus regional weights $\omega_r = 1$ for $r = 1, 2, 3, 4$.

Figure 8 USSR and Eastern Europe

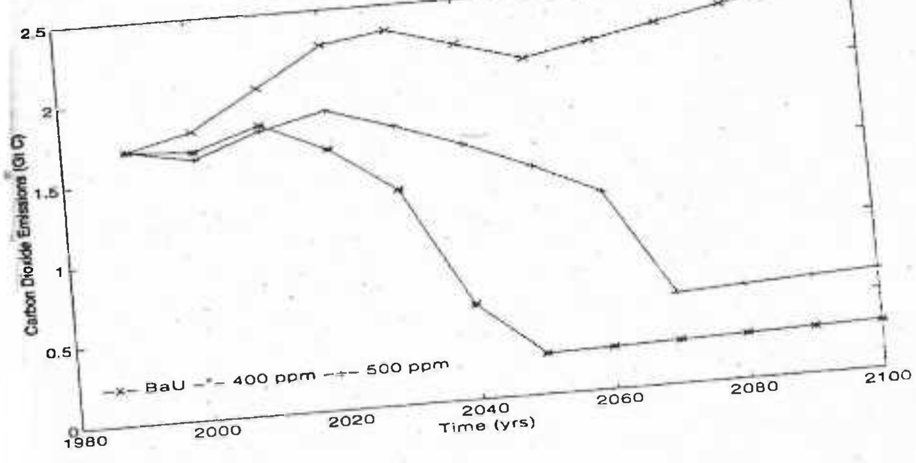


Figure 9 China and centrally planned Asia

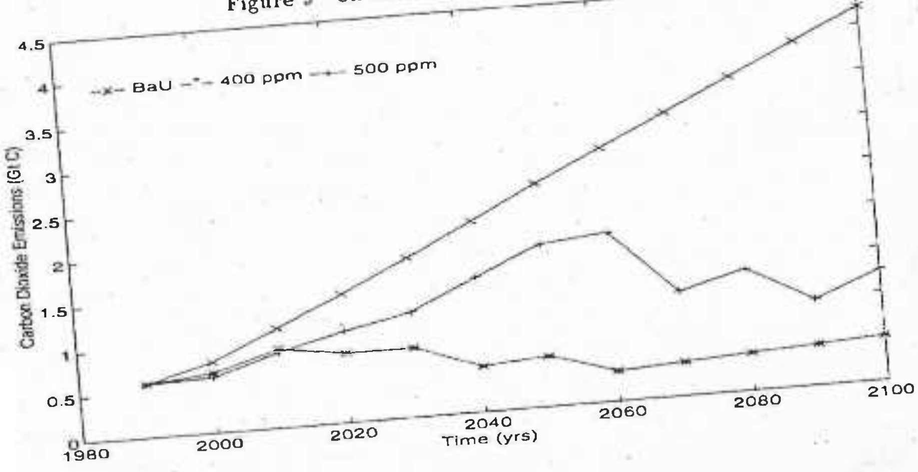
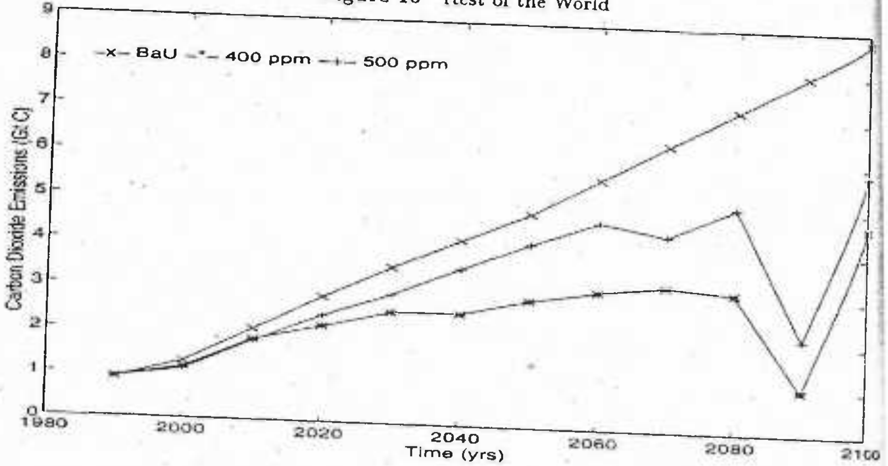


Figure 10 Rest of the World



8 CONCLUSIONS

Perhaps, the most significant finding of the preliminary analysis is the demonstration of how sensitive the optimal emission reduction strategies are to the environmental targets on CO_2 concentrations. For instance, the target of $\rho CO_2 = 400$ ppm requires emission reductions in the year 2020 of 19.0% for OECD, 28.7% for the former USSR and Eastern Europe, 45.5% for China and centrally planned Asia and 23.2% for the Rest of the World. On the other hand, the target of $\rho CO_2 = 500$ ppm in the year 2020 requires emission reductions of 12.0% for OECD, 18.1% for the former USSR and Eastern Europe, 28.6% for China and centrally planned Asia and 14.6% for the Rest of the World. Arguably, the first set of reductions is so severe as to be very unlikely to be agreed upon and implemented. Furthermore, under the 500 ppm CO_2 concentration target the truly draconian cuts in emissions may be postponed until after the year 2050, and in all the four regions of the world.

Equally importantly, perhaps, we may have demonstrated that optimization techniques can provide a useful tool to the environmental policy analyst.

APPENDIX A

IMAGE 1.0 FORMULATION

The mathematical representation of IMAGE 1.0 (Braddock et. al., 1994 and Zapert, 1994) takes the form:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x}(t) = F(\mathbf{x}(t)) + \mathbf{u}(t) \tag{A.1}$$

$$\mathbf{x}(t_0) = \mathbf{x}_0$$

where the time $t \in [1990, 2100)$, $t_0 = 1990$ is the initial simulation time, $\mathbf{x}(t) \in \Re^{155}$ is a vector of state variables, $\mathbf{u}(t) \in \Re^{155}$ is the forcing term or human interference term, and $F(\mathbf{x}(t)) : \Re^{155} \rightarrow \Re^{155}$ describes the climate system processes. The solution $\mathbf{x}(t)$ as a function of time is a trajectory of equation (A.1).

In representing the system components, the state vector $\mathbf{x} = \mathbf{x}(t) \in \Re^{155}$ at time t, is partitioned into the following variable groups.

$$\mathbf{x} = [\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \mathbf{x}^{(3)}, \mathbf{x}^{(4)}, \mathbf{x}^{(5)}, \mathbf{x}^{(6)}, \mathbf{x}^{(7)}, \mathbf{x}^{(8)}]$$

where

$\mathbf{x}^{(1)} = (x_1, \ldots, x_{12}) \in \Re^{12}$	ocean carbon module representing the amount of carbon in the twelve ocean layers.
$\mathbf{x}^{(2)} = (x_{13}) \in \Re$	concentration of carbon dioxide in the atmosphere.
$\mathbf{x}^{(3)} = (x_{14}, \ldots, x_{62}) \in \Re^{49}$	amount of carbon in the seven levels of the seven ecosystems.
$\mathbf{x}^{(4)} = (x_{63}, \ldots, x_{69}) \in \Re^{7}$	areas of the seven ecosystems of the land use module.
$\mathbf{x}^{(5)} = (x_{70}, \ldots, x_{91}) \in \Re^{22}$	ecosystem area transfer rates.
$\mathbf{x}^{(6)} = (x_{92}, \ldots, x_{140}) \in \Re^{49}$	temperature change of the 49 layer ocean module w.r.t. 1990.
$\mathbf{x}^{(7)} = (x_{141}) \in \Re$	temperature change of the mixed ocean - atmospheric layer.
$\mathbf{x}^{(8)} = (x_{142}, \ldots, x_{155}) \in \Re^{14}$	greenhouse gases, $x_{142}, \ldots, x_{151}, x_{152}, x_{153}$, and x_{154} represent concentrations of CFCs, CO, CH_4, and N_2O, respectively. x_{155}, represents OH production radicals.

The forcing term $\mathrm{u}(t)$ is a 155-vector partitioned according to the state vector partitioning and represents the human interference in the system (A.1). The nonzero components of $\mathrm{u}(t)$ are associated with blocks $x^{(2)}, x^{(5)}$ and $x^{(8)}$. Therefore

$$\mathrm{u} = \left[0, \mathrm{u}^{(2)}, 0, 0, \mathrm{u}^{(5)}, 0, 0, \mathrm{u}^{(8)}\right]$$

where

$\mathrm{u}^{(2)} = (u_{13} \in \Re$	emission of carbon dioxide from anthropogenic sources.
$\mathrm{u}^{(5)} = (u_{70}, \ldots, u_{91}) \in \Re^{22}$	ecosystem transfer rates, the amount of land per year allocated from one ecosystem to another.
$\mathrm{u}^{(8)} = (u_{142}, \ldots, u_{155}) \in \Re^{14}$	other than carbon dioxide greenhouse gas emissions.

Implicitly, the forcing term $\mathrm{u}(t)$ includes the given scenario's future population growth, fossil fuel combustion, deforestation, and the technology development in the period 1990-2100 (Braddock, et. al., 1994, and Zapert, 1994).

APPENDIX B

DICE FORMULATION

Primarily an economic optimization model with highly simplified climate dynamics DICE (Nordhaus, 1992, 1993 and 1994), calculates optimal trajectories for both *capital accumulation* and *greenhouse gas emission reductions* by maximizing a discounted value of "utility" or satisfaction from consumption subject to economic and geophysical constraints. The subset of the equations of DICE that is used in our implementation is described below. The criterion to be maximized is:

$$f(z_1(t), z_2(t)) = \sum_{t=1990}^{2100} L(t)(\log(c(t))(1+\rho)^{(1990-t)} \tag{B.1}$$

subject to

$$c(t) = \frac{(1 - z_2(t))Q(t)}{L(t)} = \frac{(1 - z_2(t))}{L(t)} \left\{ \frac{1 - b_1(z_1(t))^{b_2}}{1 + \theta_1(T(t))^{\theta_2}} A(t)K(t)^{\gamma} L(t)^{1-\gamma} \right\} \tag{B.2}$$

$$A(t) = (1 + \delta_A(t))A(t-1) \tag{B.3}$$

$$\delta_A(t) = \delta_A(t-1)(1 - \zeta_A) \tag{B.4}$$

$$L(t) = (1 + \delta_p(t))L(t-1) \tag{B.5}$$

$$\delta_p(t) = \delta_p(t-1)(1 - \zeta_p) \tag{B.6}$$

$$\alpha(t) = (1 - \delta_\alpha(t))\alpha(t-1) \tag{B.7}$$

$$\delta_\alpha(t) = \delta_\alpha(t-1)(1 - \zeta_\alpha) \tag{B.8}$$

$$K(t) = (1 - \delta_K)K(t-1) + z_2(t)Q(t) \tag{B.9}$$

$$u_{13}(t) = (1 - z_1(t))\alpha(t)Q(t) \tag{B.10}$$

$$T(t) = \nu(\mathrm{x}(t)) \tag{B.11}$$

where

$$Q(t) = \frac{1 - b_1(z_1(t))^{b_2}}{1 + \theta_1(T(t))^{\theta_2}} A(t)K(t)^{\gamma} L(t)^{1-\gamma}.$$

The economic dynamics component from DICE incorporated into OMEGA (see (1.7)) consists of equations (B.3)- (B.9). Of course, the objective function in Stage 1 is (B.1) with $c(t)$ replaced by (B.2). All the remaining variables are supplied by IMAGE 1.0.

Temperature transformation equation

The following equation is used to transform the *temperature of the mixed ocean layer* $x_{141}(t)$ from IMAGE 1.0 to the *surface temperature above land* required by DICE (Janssen, 1995):

$$T(t) = \frac{f \cdot RF(t) - k \cdot x_{141}(t)}{f \cdot \lambda + k}. \tag{B.12}$$

Decision Variables

$z_1(t)$ the rate of emission reductions of greenhouse emissions
$z_2(t)$ the rate of investment in tangible capital

Variables

$c(t)$	flow of consumption per capita at time t.
$L(t)$	level of population at time t.
δ_P	rate of population growth at time t.
ζ_P	rate of decline in population growth.
ρ	pure rate of social time preference ($\rho = 0.03\%$ per year).
$A(t)$	level of technology at time t.
δ_A	rate of growth of total productivity at time t.
ζ_A	rate of decline in productivity growth.
$K(t)$	capital at time t.
δ_K	rate of depreciation of the capital stock, 10% per annum.
γ	elasticity of output with respect to capital taken as 0.25.
$\alpha(t)$	rate of decarbonization at time t.
$\delta_\alpha(t)$	rate of growth of α at time t.
ζ_α	rate of decline in δ_α.
θ_1, θ_2	scale and nonlinearity in the damage function.
b_1, b_2	scale and nonlinearity in the cost function.
$T(t)$	increase in the average temperature in the atmosphere and upper level of the
f	fraction of the world covered by land = 0.3.
$RF(t)$	total change in radiative forcing at time t.
k	heat transfer between land and oceans.
$x_{141}(t)$	temperature of the mixed ocean layer at time t.
λ	climate sensitivity factor.
$u_{13}(t)$	emissions of greenhouse gases at time t.
$Q(t)$	gross world product at time t.

APPENDIX C

COUNTRIES CONTAINED WITHIN EACH OF THE FOUR REGIONS

OECD (Organization for Economic Co-operation and Development)

There are 24 full-member countries of the OECD, former Yugoslavia, though having some association with some OECD activities, is not included. The OECD essentially comprises the developed industrial nations operating a market economy. Most of the 24 OECD countries are in the World Bank's high-income group, the exceptions being

Greece, Portugal and Turkey, which are classed as middle-income. OECD countries are as follows:

Australia	Austria	Belgium	Canada	Denmark
Finland	France	Germany	Greece	Iceland
Ireland	Italy	Japan	Luxembourg	Netherlands
New Zealand	Norway	Portugal	Spain	Sweden
Switzerland	Turkey	U.K.	U.S.A.	

Former USSR & Eastern Europe

The former USSR and all Eastern European countries which have, or had prior to 1990, communist regimes and directed or planned economies. All of these countries are qualified as middle income economies and are as follows:

Albania	Bulgaria	Czechoslovakia	Hungary
Poland	Romania	USSR	Yugoslavia

China & Centrally Planned Asia

These countries are largely low income countries with directed or planned economies. Burma is included since its economic and social characteristics are still largely determined by its past centrally directed economy. The region is dominated by China.

Burma	Cambodia	China	North Korea
Laos	Mongolia	Vietnam	

Rest of the World

The rest of the world region takes includes the Asia Pacific, South Asia, Africa, Latin America and the Carribbean. The Asia Pacific countries included are all market economies. Most of which are middle income except for Brunei, Hong Kong and Singapore which are high income countries. Indonesia is classed as a borderline between low and middle income. South Asian countries include in Indian sub-continent and adjacent countries. All members are classified as having low income economies. The African region contained low, middle and a few high income economies. Cyprus and Malta are include here. The majority of countries in the Latin America and Carribean groups rank as middle income. However, Haiti and Guyana are low income, and the Bahamas and Bermuda are high-income.

Asia Pacific

Brunei	Fiji	Hong Kong	Indonesia
South Korea	Macao	Malaysia	Papua NG
Philippines	Singapore	Taiwan	Thailand

South Asia

Afghanistan	Bangladesh	Bhutan	India
Nepal	Pakistan	Sri Lanka	

Africa

Algeria	Angloa	Bahrain	Benin
Botswana	Burkina Faso	Burundi	Cameroon
CAR	Chad	Congo	Cote d'Ivöire
Cyprus	Egypt	Ethiopia	Gabon
Ghana	Guinea	Iran	Iraq
Israel	Jordan	Kenya	Kuwait
Lebanon	Lesotho	Liberia	Libya
Madagascar	Malawi	Mali	Malta
Mauritania	Mauritus	Morocco	Mozambique
Nambia	Niger	Nigeria	Oman
Qatar	Rwanda	Saudi Arabia	Senegal
Sierra Leone	Somalia	South Africa	Sudan
Syria	Tanzania	Togo	Tunisia
UAE	Uganda	North Yemen	South Yemen
Zaire	Zambia	Zimbabwe	

Latin A. / Carib

Argentina	Bahamas	Barbados	Bermuda
Bolivia	Brazil	Chile	Colombia
Costa Rica	Cuba	Dominican Rep	Ecuador
El Salvador	Guatemala	Guyana	Haiti
Honduras	Jamaica	Mexico	Neth Antilles
Nicaragua	Panama	Paraguay	Peru
Puerto Rico	Trinidad & Tob	Uruguay	Venezuela

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REFERENCES

[1] Braddock, R.D., J.A. Filar, R. Zapert, M.G. den Elzen and J. Rotmans, " Mathematical Formulation of the IMAGE Greenhouse Effect Model", Applied Mathematical Modelling, 1994, Vol. 18, pp. 234 – 254.

[2] Brent, R.P., "Algorithms for Minimization without Derivatives", Prentice-Hall series in Automatic Computation, 1973.

[3] Burden, R.L. and J.D. Faires, "Numerical Analysis", fourth edition, PWS-KENT, 1989, pp. 251 – 257.

[4] Economist Books, "The Economist Book of Vital World Statistics", Introduction by R.J. Samuelson, Time Books, 1990, pp. 16 – 17.

[5] Fletcher, R., "Practical Methods of Optimization", Vol. 1, Unconstrained Optimization, and Vol. 2, Constrained Optimization, John Wiley and Sons, 1980.

[6] Foulds, L.R., "Optimization Techniques: An Introduction", Springer-Verlag, New York, 1981.

[7] Gill, P.E., W. Murray and M.H. Wright, "Practical Optimization", Academic Press, London, 1981.

[8] Hoel, M., "International Environmental Conventions: The case of uniform reductions of emissions", Environmental and Resource Economics, 1992, Vol. 2, pp. 141 – 159.

[9] Horst, R. and P.M. Pardalos (Editors), "Handbook of Global Optimization", Nonconvex Optimization and Applications Vol. 2, Kluwer Academic Publishers, 1995.

[10] IPCC (Intergovernmental Panel on Climate Change), "Climate Change: The IPCC Response Strategies", Edited by J.T. Houghton, Island Press, 1991.

[11] IPCC (Intergovernmental Panel on Climate Change), "Climate Change: The Supplementary Report to the IPCC Scientific Assessment", Edited by J.T. Houghton, B.A. Callander and S.K. Varney, Cambridge University Press, 1992.

[12] Janssen, M.A., "Optimization of a Non-linear Dynamical System for Global Climate Change", GLOBO working paper GWP 95-1, National Institute of Public Health and Environmental Protection, Bilthoven, The Netherlands, 1995.

[13] Janssen, M.A., J. Rotmans and M.G.J. den Elzen, "The Biosphysics of Integrated Assessment Models for Global Climate Change: a comparative analysis", paper prepared for the Energy Modelling Forum Working group meeting, Stanford University, USA, in prep, 1995.

[14] Luenberger, D.G. (1984), "Introduction to Linear and Nonlinear Programming", Addison-Wesley, Menlo Park, CA, USA, 1984.

[15] MATLAB, "Optimization Toolbox: For use with MATLAB", The Math Works Inc., 1992.

[16] Nemhauser, G.L., A.H.G. Rinnooy, and M.J. Todd, "Optimization", Elsevier Science Pub. Co., New York, 1989.

[17] Nordhaus, W.D., "An Optimal Transition path for controlling Greenhouse Gases", Science, 1992, vol 258, pp. 1315 – 1319.

[18] Nordhaus, W.D., "Rolling the DICE: An Optimal Transition Path for Controlling Greenhouse Gases", Resource and Energy Economics, 1993, Vol. 15, pp. 27 – 50.

[19] Nordhaus, W.D., "Managing the Global Commons: The Economics of Climate Change", MIT Press, Cambridge, MA, USA, 1994.

[20] Press, W.H., B.P. Flannery, S.A. Teukolsky, and W.T. Vetterling, "Numerical Recipes in C: The Art of Scientific Computing", Cambridge University Press, Cambridge, 1988.

[21] Ramsey, F.P., "A Mathematical Theory of Saving", The Economic Journal, Dec 1928, pp. 543 – 559.

[22] Rotmans, J., "IMAGE: An Integrated Model to Assess the Greenhouse Effect", Ph.D. thesis, Kluwer Academic Press, Dordrecht, The Netherlands. 1990.

[23] Tahvonen, O., H. von Storch and J. von Storch, "Economic Efficiency of CO2 Reduction Programs", Max-Planck-Institut fur Meteorolgie, Report No. 105, Hamburg, 1993.

[24] Zapert, R., "Uncertainty Analysis of Enhanced Greenhouse Effect Models", Ph.D. Thesis, Department of Mathematics, University of Maryland Baltimore County, Maryland, USA, 1994.